INTERNATIONAL BACCALAUREATE

HL Physics Internal Assessment

<u>TITLE</u>

Relationship between Point of Interruption and Period of Oscillation in an Interrupted Pendulum

<u>Research Question</u>

How does varying the Point of Interruption affect the Period of Oscillation in an Interrupted Pendulum?

Examination Session: May 2021

"While watching a chandelier swing back and forth at the Cathedral of Pisa in 1583, Galileo noticed something curious. Galileo noticed that the time period to swing through one complete cycle is independent of the amplitude through which it swings.... He timed the swings with his pulse, the only timing device at hand."

(Stinner 2007)

Introduction

It was in my 5th grade Science class where I learned that without the work of Galileo Galilei, the father of the pendulum, there would have been no *Principia.* The intuition he espoused in developing ideas about the isochronous behaviour of the pendulum evoked my amazement as a child and encouraged me to pursue physics with creativity.

It was Galileo who had pioneered the use of the interrupted pendulum to explain the principle of the conservation of mechanical energy. He placed a rigid object directly below the point of suspension of a simple pendulum and released the pendulum bob from an angle. With the pendulum finally reaching the same height as its initial position, Galileo experimentally proved that the sum of kinetic and potential energy remains constant (Zeleny 2011). Over the course of the upcoming centuries, the interrupted pendulum gained a different significance in the physics labs: it was used to calculate the minimum length at which the pendulum is interrupted, for which a mass released at an amplitude of 90° would undergo a complete circular motion around the point of interruption (Miller 2002).

Inspired by Galileo's scientific endeavour, I examined the interrupted pendulum's motion from a different lens. I realised that at smaller amplitudes, an interrupted pendulum undergoes an oscillatory motion with a mixture of periods. Subsequently, I wanted to understand how a varied point of interruption would affect the period of oscillation of an interrupted pendulum. Hence, I derived the following research question,

How does varying the Point of Interruption affect the Period of Oscillation in an Interrupted Pendulum?

Background Information

The Application of Interrupted Motions

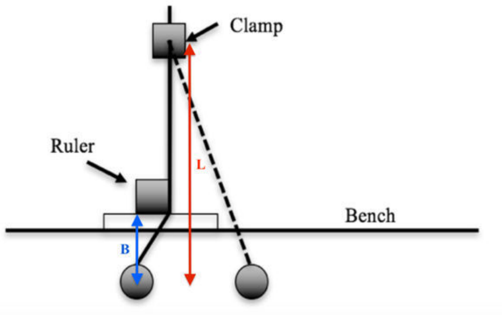
The interrupted pendulum's mixture of periods resembles the motion of many other objects in the real world. Take the motion of a 'pirate boat' ride at the amusement park for instance. As the pirate boat begins its oscillation and rises through the air, its period increases. Then, for some time, its period is constant. However, as the boat completes more swings, its period gradually decreases until it becomes stationary and the ride stops. Hence, the 'pirate boat ride' is a motion with a mixture of periods. (Beston Amusement Experience n.d.) This applies to playground swings and swaying bridges as well.

More specifically, an example of interrupted motion of a mixture of periods is that in an interrupted pendulum; it serves the purpose of making corrections to the motion of time-keeping devices when its oscillations are too fast or too slow or creating tunable time-keeping devices whose period can be adjusted by varying the point of interruption.

Mathematical Model of the Interrupted Pendulum

An interrupted pendulum refers to a simple pendulum whose swing is broken at a point directly below the point of suspension of the pendulum. The ruler serves as the point of interruption. The motion of an interrupted pendulum can be divided equally into 2 parts: before the pendulum comes into contact with the ruler and after the pendulum comes into contact with the ruler.

Figure 1: The Interrupted Pendulum



During the motion's first half, the pendulum swings with a length L (m), while during the motion's second half, the pendulum swings with a length B (m). If these two motions are looked at separately, their periods for small angles from the vertical is,

$$T_1 = 2\pi\sqrt{\frac{L}{g}} \text{ and } T_1 = 2\pi\sqrt{\frac{B}{g}}$$

where T_1 is the time period for the first uninterrupted half of the pendulum's motion (s) and T_2 is the time period for the second interrupted half of the pendulum's motion (s).

The motion is divided into two halves. Hence, T, the total time period (s) of the interrupted pendulum is,

$$T = \frac{T_1}{2} + \frac{T_2}{2} = \pi\left(\sqrt{\frac{L}{g}} + \sqrt{\frac{B}{g}}\right) = \frac{\pi}{\sqrt{g}}(\sqrt{B} + \sqrt{L})$$

Furthermore, since the length B is unconditionally lesser than the length L, the period of the first half of oscillation is always greater than that of the second half. Hence, a dimensionless relationship between the period of a simple and interrupted pendulum can be shown as below, (Miller 2002)

$$\frac{T_{interrupted\ pendulum}}{T_{simple\ pendulum}} = \frac{\frac{\pi}{\sqrt{g}}(\sqrt{B} + \sqrt{L})}{\frac{2\pi}{\sqrt{g}}(\sqrt{L})} = \frac{1 + \sqrt{\frac{B}{L}}}{2}$$

Hypothesis

As the length B, increases, the total time period of the interrupted pendulum, T, increases. As observed from the equation, $T = \frac{\pi}{g}(\sqrt{B} + \sqrt{L})$, if T is plotted against $\sqrt{B}$, and all other variables (g, L) are kept constant, a linear graph of positive gradient between T and $\sqrt{B}$ will be produced.

Preliminary Work

Pendulum Bob Mass selection: A copper mass of 0.033 kg and radius of 0.01 m was chosen as the pendulum bob because it was sufficient in pulling the string of the pendulum taut and stabilizing the pendulum motion as it swings. Additionally, since its size was small, it minimized the Magnus effect due to the spin of the pendulum- the Magnus Effect is proportional to the cube of the spinning object's radius. The Magnus Effect is experienced by a spinning object moving through a fluid, due to the difference in fluid pressure on opposite sides of the fluid. (SeattleU: College of Science and Engineering n.d.) While a topspin produces a downward force that decreases the time period, a backspin produces an upward lifting force that increases the time period of the ball.

Length selection: The total length of 0.800 m was selected because it enabled data collection for seven points of interruption with a 0.100 m difference of distance from the bob's centre of mass - (0.100, 0.200, 0.300, 0.400, 0.500, 0.600, 0.700, 0.800 m from the bob's centre of mass). This provided ample data points to plot a trend-line. A higher total length was not chosen as it would cause spinning of the mass.

***String selection*:** A string of 0.50 mm thickness was chosen because it would experience a low value of air resistance against drafts in the air-conditioned room and reduce the possibility of damped oscillations. Additionally, a thin string would not move the position of the ruler upon hitting it, thus serving to maintain the position of the ruler directly below the point of suspension of the pendulum.

***Angle selection*:** A small initial angle of 20° was chosen because at larger angles, the pendulum's behaviour would be non-harmonic and non-isochronous i.e., the period would be dependent on the oscillation's initial angle. Hence, through the selection of small-angles, it is ensured that the initial angle does not interfere with the role that the length *B* plays on the period of oscillation.

Apparatus

Apparatus	Quantity	Absolute Uncertainty	Percentage Uncertainty
Metre Rule (Lowest length *B* measured is 0.100 m)	1	± 0.001 m	$= \frac{0.001}{0.100} \times 100\% = 1\%$
Point of Interruption - Metre rule (Lowest length *B* measured is 0.100 m)	1	± 0.0025 m (half the value of its thickness of 5 mm)	$= \frac{0.0025}{0.100} \times 100\% = 2.5\%$
Stopwatch (Lowest time measured is 12.37 s)	1	± 0.20 s for human reaction time	$= \frac{0.20}{12.37} \times 100\% = 1.62\%$
Protractor	1	$\pm 0.5°$	$= \frac{0.5}{20} \times 100\% = 2.5\%$
Clamp	2	-	-
Retort Stand	2	-	-
Bench	1	-	-
Pendulum Bob	1	-	-
Total Percentage Uncertainty			7.62%

Table 1: Apparatus List with Absolute Uncertainty and Percentage Uncertainty.

Experimental Design

The total length of the pendulum is kept constant at 0.800 m, and the lengths of the pendulum at where the point of interruption is placed was varied. The time for 10 oscillations of the pendulum will be measured using a Stopwatch. A metre rule will be used as the point of interruption, as it has a sharp edge. However, its finite thickness contributes to the percentage uncertainty of measured *B* (m).

The length *L* is measured between the point of suspension and the pendulum bob's centre of mass. The length *B* is measured between the point of interruption and pendulum bob's centre of mass - (0.100, 0.200, 0.300, 0.400, 0.500, 0.600, 0.700, 0.800 m from the bob's centre of mass).

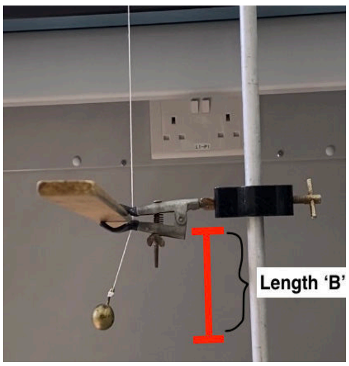


Figure 2.a: Interrupted Section Set-up

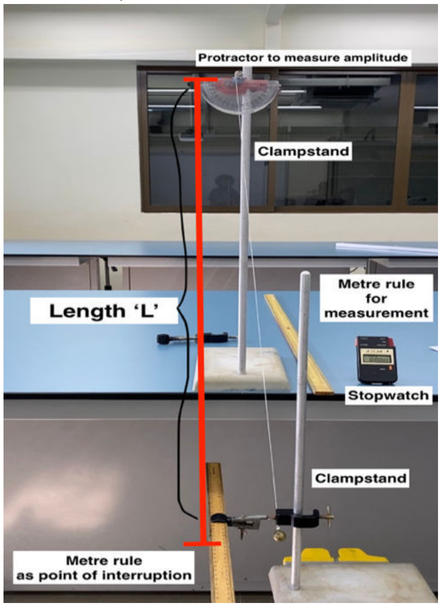


Figure 2.b: Overview of the Set-up

Experimental Variables

	Variable	Method of Measurement
Independent	Length *B*, the distance between the point of interruption and pendulum bob's centre of mass	The point of interruption is placed at 0.100 m intervals from the pendulum bob's centre of mass, starting at 0.100 m to 0.800 m. The final length of *B* equates to the length *L*, i.e. no point of interruption- lengths are measured using a metre ruler.
Dependent	The time taken for the interrupted pendulum to complete 10 oscillations	A stopwatch measured the time taken for the pendulum to complete 10 oscillations. The experimenter judged when the pendulum began and finished 10 oscillations. The number of oscillations was counted manually as the 1^{st}, 2^{nd}, 3^{rd}, etc.

Table 2: Independent and Dependent Variables, and Method of Measurement

Control Variable	Reason for Control	Method of Control
The total length of the pendulum string, *L*	The total length of the string affects the period of the pendulum; the square of time period is directly proportional to the string length of the pendulum, as outlined in the formula, $T = 2\pi\sqrt{l/g}$. Longer strings cause the pendulum to fall further, leading to lower frequency and higher time period.	Maintaining the length of the string constant at 0.800 m.
Choice of metre ruler (for point of interruption)	The finite thickness of the ruler implies that the point of interruption has an uncertainty of $\pm 1/2$ of the ruler's thickness. Thicker rulers imply larger uncertainties for the values of *B*, since its measured from the point of interruption.	Using the same edge of the ruler of thickness 5 mm.
String material	The string material determines the extent to which the string extends under the pendulum bob's weight; the string's extension must be negligible to prevent length *L* from changing.	Using inextensible nylon string of thickness approximately 0.5 mm.
Amplitude of oscillation	To ensure that the motion is conducted under small-angles, so non-isochronous behaviour is retained.	Using a protractor to maintain the initial swinging angle at 20°.

Table 3: Controlled Variables, Reason for Control, and Method of Control

Experimental Procedure

1. Tie one end of the string to the clamp stand with the other end securely fastened with a metallic pendulum bob.
2. Measure the length of the pendulum to be 0.800 m from the pendulum bob's centre of mass.
3. Displace the ball gently from its equilibrium position by 20°, which is measured using a protractor, then release it without imparting any impulse on it.
4. Start the stopwatch.
5. Count 10 oscillations manually.
6. Stop the stopwatch after 10 oscillations.
7. Note the time taken to complete 10 oscillations.
8. Move the point of interruption 0.700 m, 0.600 m, 0.500 m, 0.400 m, 0.300 m, 0.200 m and 0.100 m from the pendulum bob's centre of mass. These distances are known as length *B*.
9. Repeat steps 1 to 7 five times for each length *B*.

Five Experimental Assumptions

Assumption	Reality	Best Way the Assumption was Fulfilled
The string of the pendulum is massless and always remains taut.	There are two ways of ensuring the tautness of a string: using an infinitely thick string or using an infinitely stiff string of negligible strain. However, while the former would not be of low mass, the latter wouldn't have the ability to be interrupted.	It was ensured that the mass of the string is a small fraction of that of the pendulum bob. In this investigation, a string made of low-density material of nylon was chosen. This way, the low-mass string was stretched taut by the heavy copper mass.
The pendulum bob acts as a point mass. Hence, it will experience negligible Magnus Effect (proportional to the cube of a spinning object's radius).	The pendulum bob isn't a point mass; hence, it will always be the case that the pendulum bob will experience the Magnus effect. As the pendulum bob experiences either a topspin or backspin, the period will either decrease or increase.	A small pendulum bob of diameter many times smaller than the total length of the pendulum was used: the diameter of the pendulum bob was approximately 40 times lesser than the length L. Hence, the pendulum bob approximately behaved as a point mass.
There is negligible drag along the pendulum string or bob.	The motion loses energy due to the drag force of air resistance, thus increasing the period of motion.	The effect of the drag force was reduced by dropping the pendulum from small initial heights with low initial velocities. Low radius of the selected pendulum bob decreased experienced drag force. (Drag Force= $6\pi r\eta \upsilon$, where r is bob radius, η is fluid viscosity and υ is object velocity).
The motion of the pendulum is only to-and-fro; The pendulum doesn't move torsionally.	Friction between the edge of the ruler and string exerts a torque onto the string, imparting a torsional force onto it. As a result, the pendulum bob spins.	Care was taken to release the pendulum bob from a point parallel to its point of suspension, with no initial velocity, to minimise its spin.
The clamp stand and point of interruption does not move. This is to ensure that the point of interruption is located directly below the point suspension of the pendulum bob, so that the period of the interrupted pendulum can be divided equally into two.	Every time the pendulum bob hits the ruler, the ruler moves by a miniscule amount.	A heavy object was placed on the ruler, so that it doesn't move.

Table 4: Experimental Assumptions, Reality, and Best Way to Fulfil the Assumption

Raw Data

	For 10 oscillations					
Amplitude /±0.5°	**Length B/ ± 0.001 m**	T_1/ ±0.01 s	T_2/ ±0.01 s	T_3/ ±0.01 s	T_4/ ±0.01 s	T_5/ ±0.01 s
20.0	0.100	12.56	12.43	12.47	12.37	12.59
20.0	0.200	13.91	13.87	13.91	13.94	13.91
20.0	0.300	14.81	14.75	14.81	14.87	14.82
20.0	0.400	15.81	15.72	15.94	15.97	15.59
20.0	0.500	16.69	16.56	16.63	16.35	16.50
20.0	0.600	17.09	17.00	17.15	17.16	17.13
20.0	0.700	17.59	17.84	17.57	17.69	17.62
20.0	0.800	18.31	18.50	18.06	18.41	18.40

Table 5: Five Trials of Measured Time Taken for 10 oscillations at different lengths B, from 0.100 cm to 0.800 cm, at 0.100 intervals. Initial angle of swing kept constant at 20.0°.

Processed Data

	For 10 oscillations						For 1 oscillation
Length B/ ± 0.001 m	T_1/ ±0.01 s	T_2/ ±0.01 s	T_3/ ±0.01 s	T_4/ ±0.01 s	T_5/ ±0.01 s	$T_{average}$/ s	T /s
0.100	12.56	12.43	12.47	12.37	12.59	12.48 ± 0.11	1.25 ± 0.01
0.200	13.91	13.87	13.91	13.94	13.91	13.91 ± 0.04	1.39 ± 0.00
0.300	14.81	14.75	14.81	14.87	14.82	14.81 ± 0.06	1.48 ± 0.01
0.400	15.81	15.72	15.94	15.97	15.59	15.81 ± 0.19	1.58 ± 0.02
0.500	16.69	16.56	16.63	16.35	16.50	16.55 ± 0.19	1.65 ± 0.02
0.600	17.09	17.00	17.15	17.16	17.13	17.11 ± 0.19	1.71 ± 0.01
0.700	17.59	17.84	17.57	17.69	17.62	17.66 ± 0.19	1.77 ± 0.01
0.800	18.31	18.50	18.06	18.41	18.40	18.34 ± 0.19	1.83 ± 0.02

Table 6: Average Time Taken for 10 oscillations with uncertainty, as well as Time Period with uncertainty. Calculated from Raw Data.

The following formula was used to calculate the time period of an interrupted pendulum. A worked example is shown for Length B, 0.1000 m.

For 10 oscillations,

$$T_{average} = \frac{T_1 + T_2 + T_3 + T_4 + T_5}{5} = \frac{12.56 + 12.43 + 12.47 + 12.37 + 12.59}{5} = 12.48 \text{ s}$$

Following this, for 1 oscillation,

$$T = \frac{T_{average}}{10} = \frac{12.48}{10} = 1.25 \text{ s}$$

The following formula was used to calculate the absolute uncertainty of the time period. A worked example is shown for Length B, 0.10000 m.

For 10 oscillations,

$$\Delta T_{average} = \frac{T_{max} - T_{min}}{2} = \frac{12.59 - 12.37}{2} = 0.11$$

For 1 oscillation,

$$\Delta T = \frac{\Delta T_{average}}{10} = \frac{0.11}{10} = 0.011$$

The following formula was used to calculate the uncertainty of √B.
A worked example is shown for Length *B*, 0.10000 m. Uncertainties for rest of the values of *B* are shown in Table 7.

$$Uncertainty\ of\ \sqrt{B} = \frac{1}{2} \times \frac{\Delta B}{B} \times \sqrt{B} = \frac{1}{2} \times \frac{0.001}{0.10000} \times 0.3162$$
$$= 0.00158 \approx 0.002$$

Length B/ ± 0.001 m	$\sqrt{B}$ / $\sqrt{m}$
0.100	0.316 ± 0.002
0.200	0.447 ± 0.001
0.300	0.548 ± 0.001
0.400	0.633 ± 0.001
0.500	0.707 ± 0.001
0.600	0.775 ± 0.001
0.700	0.837 ± 0.001
0.800	0.894 ± 0.001

Table 7: Length B and Square Root of Length B with Absolute Uncertainty.

The following formula was used to calculate the standard deviation of the time taken for 10 oscillations.
A worked example is shown for Length B, 0.10000 m. Uncertainties for rest of the values of *B* are shown in Table 8.

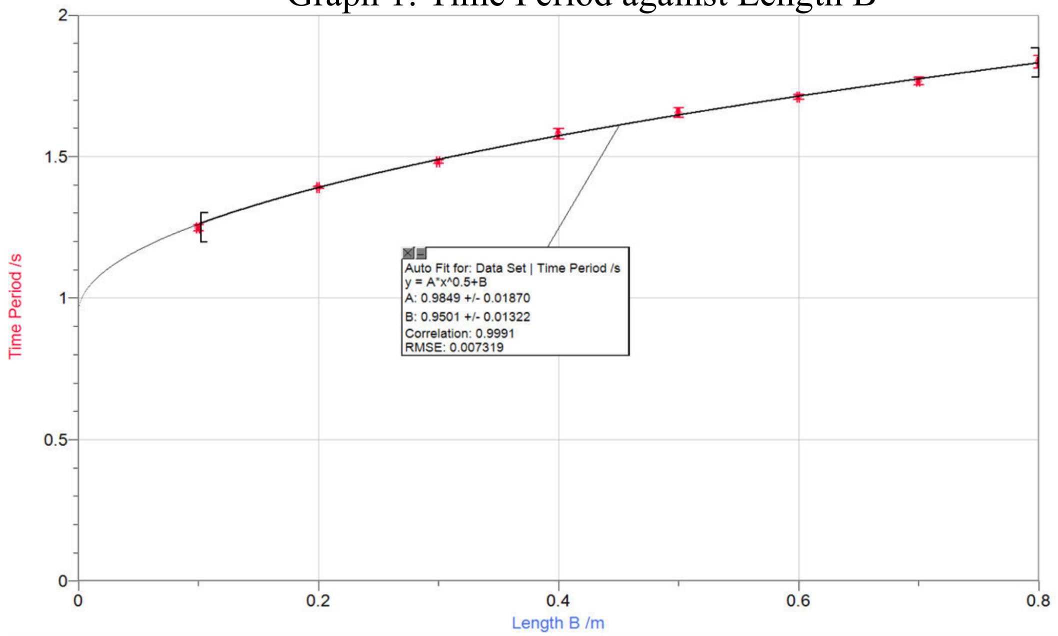
$$\sigma_T = \sqrt{\frac{\sum(T - \bar{T})^2}{5}}$$
$$= \sqrt{\frac{(12.56 - 12.48)^2 + (12.43 - 12.48)^2 + (12.47 - 12.48)^2 + (12.37 - 12.48)^2 + (12.59 - 12.48)^2}{5}}$$
$$= 0.08\ s$$

Length B/ ± 0.001 m	σ_T/s
0.100	0.08
0.200	0.02
0.300	0.04
0.400	0.14
0.500	0.12
0.600	0.06
0.700	0.10
0.800	0.15

Table 8: Standard Deviation of Time Taken for 10 oscillations at different values of Length B

Graphs

Graph 1: Time Period against Length B



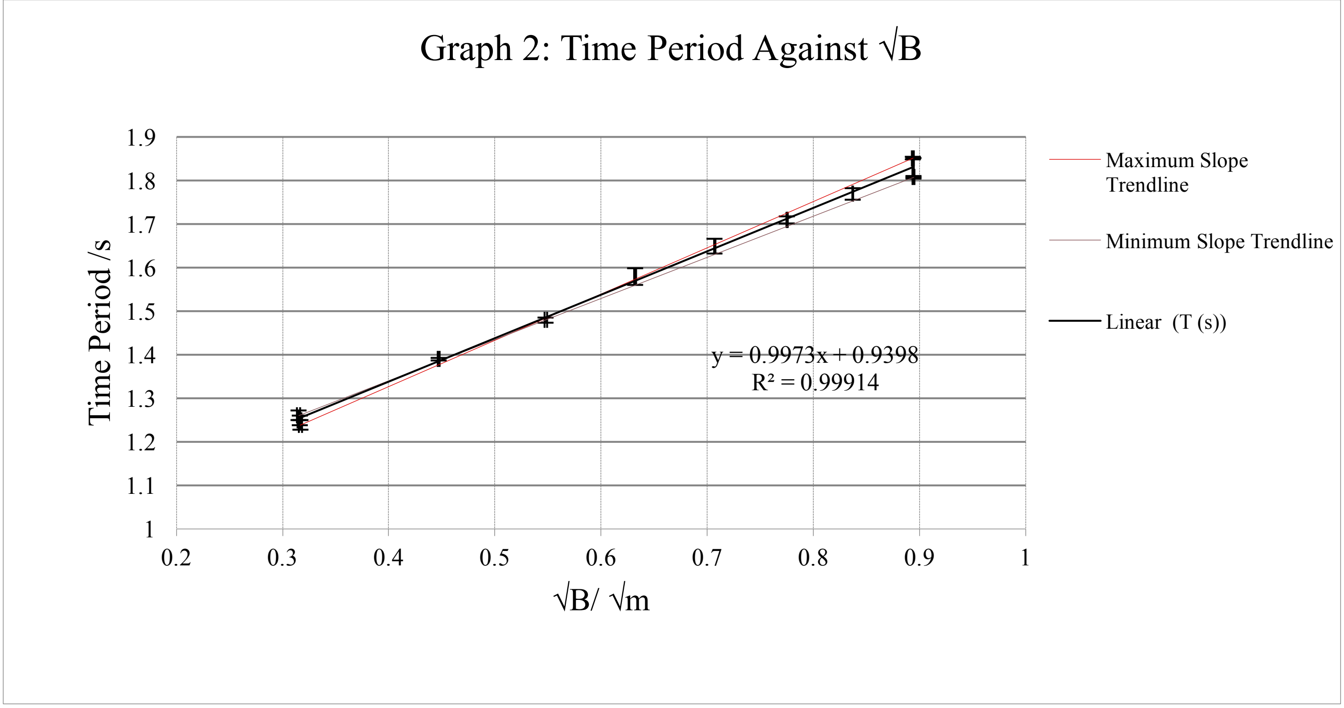
The regression line of Graph 1 defines the following square root function,

$$T = a\sqrt{B} + b$$

where a and b are constants.

a and b were found to be as follows,

$$a = (0.985 \pm 0.019)$$
$$b = (0.950 \pm 0.013)$$



Maximum slope Trend-line	**Minimum slope Trend-line**	**Uncertainty of gradient**	**Uncertainty of intercept**
y= 1.06x+0.902	y=0.945x+0.963	$\frac{1.06-0.945}{2}$ =0.0575	$\frac{0.963-0.902}{2}$ =0.0305

Line of Best Fit: y = 0.9973x + 0.9398

The relationship is therefore,

$$T = (0.997 \pm 0.058)\sqrt{B} + (0.940 \pm 0.031) \quad [1]$$

Analysis and Conclusion

Comparing theoretical and experimental values

The theoretical formula for the motion of the interrupted pendulum is $T = \pi\left(\sqrt{\frac{L}{g}} + \sqrt{\frac{B}{g}}\right) = \frac{\pi}{\sqrt{g}}\sqrt{B} + \pi\sqrt{\frac{L}{g}}$

Hence, for a T vs. $\sqrt{B}$ graph, the theoretical gradient is $\frac{\pi}{\sqrt{g}} \approx 1.00$ and the theoretical y-intercept is $\pi\sqrt{\frac{L}{g}} = \pi\sqrt{\frac{0.800}{g}} \approx 0.897$.

Comparing the theoretical formula to the experimental relationship between T and $\sqrt{B}$, which was derived from Graph 2 and expressed in Equation 1,

Comparing gradients	**Comparing *y*-intercepts**
The experimental gradient is (0.997 ± 0.058) compared to a theoretical gradient of 1.00.	The experimental y-intercept is (0.940 ± 0.031) compared to a theoretical y-intercept of 0.897

The theoretical gradient is well within the range of values of the experimental gradient while the theoretical intercept is below the minimum experimental value of the intercept as shown below.

Taking into account intercept uncertainty, the minimum experimental value of the y-intercept is $0.940 - 0.031 = 0.909$. However, the theoretical y-intercept of 0.897 is lesser than 0.909.

Hence, it is suggested that a systematic error caused an increase in the experimental value of the intercept.

This is corroborated by the results in Graph 1. Comparing the theoretical formula to the regression function of Graph 2, we know that $a = \frac{\pi}{\sqrt{g}} \approx 1.00$ and $b = \pi\sqrt{\frac{0.800}{g}} \approx 0.897$.

Comparing a values	**Comparing b values**
The experimental a value is (0.985 ± 0.019) compared to a theoretical a value of 1.00.	The experimental b value is (0.950 ± 0.013) compared to a theoretical b value of 0.897

Even in this case, the theoretical a value is well within the range of experimental a values, while the theoretical b value is below the minimum experimental b value as shown below.

Taking into account intercept uncertainty, the minimum experimental b value is $0.950 - 0.013 = 0.937$. However, the theoretical b value of 0.897 is lesser than 0.937.

Hence, it is suggested that a systematic error caused an increase in the b value. In all, it is evident that a strong systematic error that shifted the Graph 1 and 2 upwards is present in this investigation.

Regression Analysis

From Graph 1, the coefficient of determination (R^2) is 0.9991 for a square root regression. This is indicative of a strong square root relationship between Length B and Time Period. From Graph 2, the coefficient of determination (R^2) is 0.99914 for linear regression. This is indicative of a strong linear relationship between $\sqrt{B}$ and Time Period. Hence, there is a strong positive correlation between these two variables, and we can accept the hypothesis that as the length B, increases, the total time period of the interrupted pendulum, T_{total}, increases.

Uncertainty, Error Bars and Outliers

In Graph 1, the horizontal error bars are not clearly visible as they are too small. This is because the uncertainty of B is only $\pm$ 0.001 m or approximately 1% for different values of B. On the other hand, vertical error bars are visible as the half range uncertainty of time period is comparable to the order of magnitude of the time period, due to the significant effect of human reaction time. Lastly, there are no outliers for all trials. Hence, there are no anomalies.

Standard Deviation

The standard deviation values for the time taken for 10 oscillations is presented in Table 7. The standard deviation can be attributed to any random error, from the damping of the pendulum to human reaction time. Standard deviation values are in the order of magnitude of -1 to -2, in comparison to the average time taken to complete 10 oscillations, which is in the order of magnitude of 1. Additionally, the maximum standard deviation itself is 0.15 s, which is only 0.82% of the average Time Period (18.34 s) at that Length B (0.800 m). Hence, for this Investigation, it can be understood that the standard deviation values are low and the data procured for time taken for 10 oscillations are precise. The values of standard deviation can be further lowered by increasing the number of trials taken and reducing random error.

In general, one can say that the collected data reflects a high precision but lacks accuracy, possibly caused by a systematic error in the process. Potential systematic errors are shown in Table 9. Nevertheless, the errors in the experiment do not hinder the observation of a clear trend thus allowing the acceptance of the hypothesis that as the length B, increases, the total time period of the interrupted pendulum, T_{total}, increases.

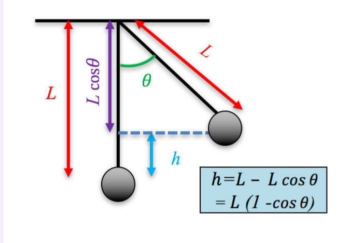
Evaluation

Source of Systematic Error	Effect on results	Improvement
Time delay at the point of interruption due to the string slipping sideways against the edge of the ruler.	Time delay adds onto the time taken for 1 oscillation and increases the period of the interrupted pendulum.	Use a thinner ruler or a nail that would decrease the time delay due to slipping.
Length *B* and *L* was measured from the geometric centre of mass of the sphere that the pendulum bob was approximated to be. However, since the pendulum bob has an attached hook, its actual centre of mass is higher than its geometric centre of mass.	This leads to larger measured values of *B* and *L* and larger time periods - they are proportional, as demonstrated by $T = \pi\left(\sqrt{\frac{L}{g}} + \sqrt{\frac{B}{g}}\right)$. However, since the hook is miniscule, this may have a negligible effect of results.	Using a metal ball that has a smaller hook attached to it would make it less erroneous to approximate the pendulum bob as a sphere.
Increased length *L* of the string due to its extension under the weight of the pendulum bob.	An increased length *L* implies more time taken for the first uninterrupted half of the pendulum's motion. Hence, larger time periods are measured for the same length *B*. However, this is of low significance as there was no qualitative observation of length increase during the experiment. Moreover, any length increase would be negligible due to the inextensible nature of the chosen nylon string.	Use new string for each trial to avoid the use of strings that were stretched and worn out by previous trials.
Certain amount of air resistance acting against the oscillatory motion of the pendulum, due to the presence of drafts in a well-ventilated room.	Air resistance can increase the experimental time periods. The effect of air resistance may increase as the velocity of the pendulum increases towards equilibrium position.	Being insignificant, no improvement needed (see calculation of air resistance below)

Table 9: List of Systematic Errors, effect on results and possible ways of improvement.

To understand the extent to which air resistance affected experimental results, it was quantified by Stokes' Law.

<u>*Stokes' Law*</u> states that, for small masses, $F = 6\pi r\eta v$ (OpenStax College n.d.) where F is air resistance (N), r is bob radius (m), η is dynamic viscosity ($\text{kgm}^{-1}\text{s}^{-1}$) and v is object velocity (ms^{-1}) For this investigation, the dynamic viscosity of air is 1.849×10^{-5} $\text{kgm}^{-1}\text{s}^{-1}$ (Engineers Edge 2000), r is 0.01 m, and v is the maximum velocity of the bob, that is used to compute the maximum air resistance. v is computed through conservation of energy calculations shown to the right.	<u>*Computing maximum velocity of the pendulum bob*</u> Potential energy at maximum positions = Kinetic energy at equilibrium $mgh = {}^{1}/_{2}\, mv^2$ Solving for v, $v = \sqrt{2gh}$, where h is the distance of the vertical pendulum bob from the equilibrium when it's at maximum position.

Therefore, Maximum $F = 6\pi \cdot 0.01 \cdot 1.849{\times}10^{-5} \cdot 0.973 =$ 3.39 μN. This is a negligible.	*Figure 3: Deriving formula for h*  Hence $h = L - L\cos\theta = 0.800 - 0.800\cos 20° =$ 0.0482 m. And $v = \sqrt{2gh} = 0.973\ \text{ms}^{-1}$

The effect of air resistance was negligible as it was quantified to be in the order of −6. Hence, the systematic error in this Investigation can be understood to be mainly due to time delay at the point of interruption. Random errors in this investigation were minimised by the repetition of trials and the use of average values for time period. However, from Table 1, we know that the total instrumental % uncertainty = total random error = 7.62%. Hence although the total random error is small, it is not negligible. Table 11 outlines potential random errors.

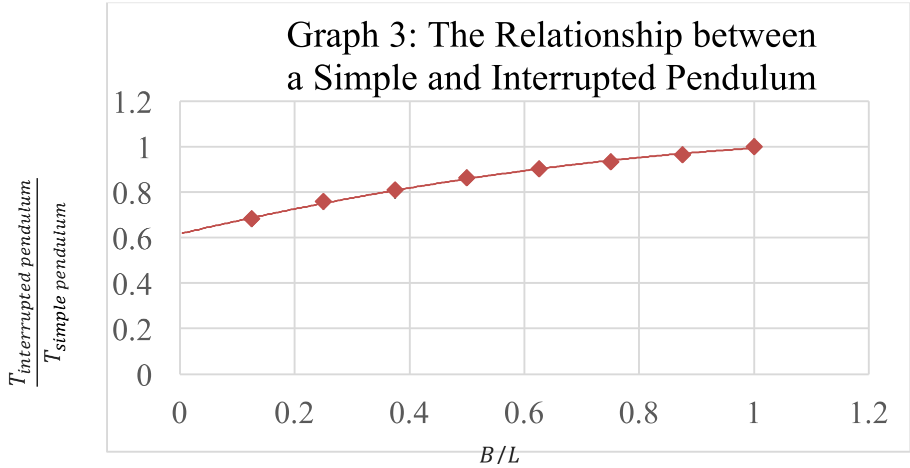
Source of Random Error	**Effect on Results**	**Improvement**
Human reaction time: pressing the start and stop button of the stopwatch too early or late.	Human reaction time leads to higher or lower measured time taken for 10 oscillations. Standard deviation values of 0.02 s-0.15 s were calculated, reflecting the range of human reaction time, 0.1 s-0.2 s. Hence, the effect of human reaction time on results is significant.	Use photogate timer to measure time taken for 10 oscillations. Alternatively, film the oscillation of the pendulum with a special camera that includes a stopwatch in it. This will allow the determination of the pendulum's time period with higher precision and diminish the vertical error bar in Graph 2.
High instrumental uncertainty of metre ruler.	It can be seen from Table 1 that the cumulative percentage uncertainty of the metre ruler as a measuring device and point of interruption contributes to almost half of the total random error (3.5%).	Use more precise instrumentation such as Vernier Calliper as a measuring device. Use a point of interruption with a thinner edge, such as a nail or thin sheet of glass.
On occasion, the pendulum moved torsionally instead of to and fro.	Increases experimental time period of one oscillation.	An electronic protractor can be used to ensure that the pendulum is released parallel to the point of suspension; it can automatically measure the angle between the point of suspension and string.
Parallax error in approximating the initial angle of swing using a protractor.	Minimal effect on results: the initial angle of the swing doesn't affect the pendulum's time period if its lower than 30°. Parallax error will only cause angle measurement to be few degrees off from 20°.	Use an electronic protractor that will measure the angle automatically, without requiring the user to read the scale. Hence, the chance of a parallax error will be removed. More effectively, one can reduce the angle of swing to around 10°, thus reducing the possibility of non-harmonic motion.

Table 10: List of Random Errors, effect on results and possible ways of improvement.

Extension: Graphing the ratio/relationship between simple and interrupted pendulum

The dimensionless relationship between period and length is given by $\frac{T_{interrupted\ pendulum}}{T_{simple\ pendulum}} = \frac{1+\sqrt{\frac{B}{L}}}{2}$.

Graph 3 illustrates the relationship between $\frac{T_{interrupted\ pendulum}}{T_{simple\ pendulum}}$ and the ratio of $B: L$.



As the point of interruption moves higher and length B approaches 0.800 m (the value of L), the time periods become equal and ratio of $B: L$ becomes 1. Consequently, the graph approaches unity. At higher lengths of B, the maximum amplitude is relatively small, and thus this motion stays within that of small-angle oscillations.

As the point of interruption moves down and length B decreases to become close to zero, the ratio of $B: L$ becomes zero. It is at this point that the curve intercepts the y-axis at approximately 0.6. The theoretical y-intercept is 0.5, as shown in the equation above the Graph 3. However, due to the systematic error where the experimental time periods are consistently higher, the ratio of $\frac{T_{interrupted\ pendulum}}{T_{simple\ pendulum}}$ is higher and so is the y-intercept. For very small lengths of B, the pendulum is observed to recoil very fast; it hits the ruler and whips around it, spending very little time on the left side of its motion. Hence, for lower lengths of B, the pendulum oscillates at large angles. For the smallest length of B, B=0.100 m, the angle after interruption is found as below,

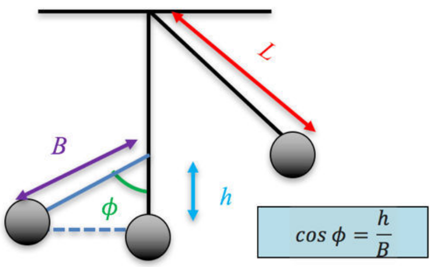


Figure 4: Angle of oscillation after interruption

> Referring to Figure 4
>
> Angle of oscillation after interruption: $\cos^{-1}\left(\frac{h}{B}\right) = \cos^{-1}\left(\frac{0.0482}{0.100}\right) = 61.2°$,
>
> a very large angle at which simple harmonic motion no longer occurs.

Therefore, at very small lengths B, the motion should be investigated further under large-angle oscillation approximations.

Safety and Precautions

Scissors were used to cut string; they were held away from fingers. No other ethical or environmental issues emerged.

Proposed Future Extension

The effect of interrupting the swing of the pendulum away from the mean position, at an angle to the point of suspension. In this case, it is uncertain that the pendulum still retains its isochronous behavior. (Gil 2003)

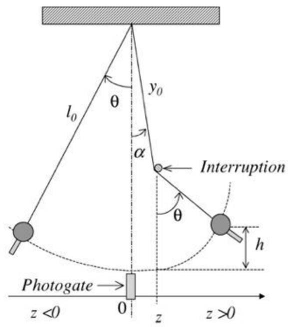


Figure 5: An interrupted pendulum, where the point of interruption is an angle, ∝, away from the line joining the point of suspension and mean position. (Gil 2003)

Figures 1 to 4 were made by the author

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