Important quantities for this topic

Constant with time for a given oscillating system	Vary with time for all oscillating systems
ω - angular velocity f - linear frequency A or X_o - Amplitude T - the time period	x - displacement (from eq_m) v - linear velocity a - linear acceleration

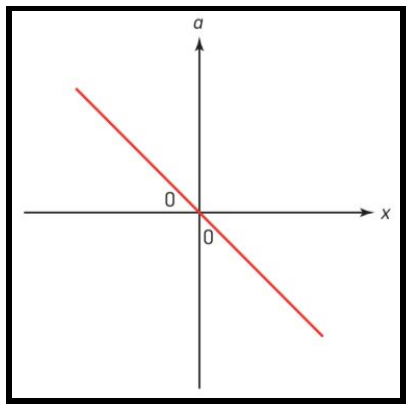
4.1 Oscillations

Oscillations are repeating vibrations. **Isochronous oscillations** are oscillations that have the same time period throughout its motion
Displacement (x) is the distance and direction from the equilibrium position (Measured in **m**)
Amplitude (A) is the maximum displacement from the equilibrium position (Measured in **m**)
Period (T) is the time taken for one complete oscillation (Measured in **s**)
Frequency is the number of oscillations in one second (Measured in **Hz**)
The following equation links frequency and time period:

$$\boxed{T = \frac{1}{f}}$$

Simple Harmonic Motion (SHM)

To perform SHM, an object must have a **restoring force** acting on it. A restoring force is a force in the opposite direction to displacement. Given below are the conditions for SHM:



- The magnitude of the force (and therefore the acceleration) is proportional to the displacement of a body from a fixed point
    - $a \propto -x$
- The minus sign before 'x' shows that acceleration always opposes the direction of displacement
    - The graph shows the line $a = -kx$. This means it has a negative gradient
- The direction of the force (and therefore the acceleration) is towards that fixed point

Phase difference

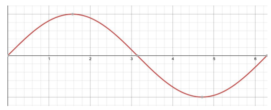
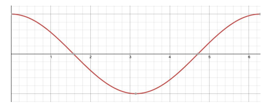
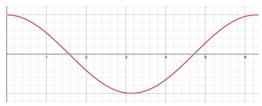
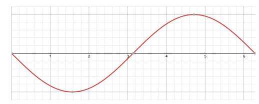
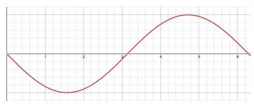
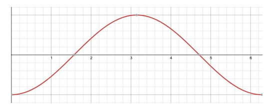
There is a similarity between the shapes of the graphs for displacement, velocity, and acceleration. All three graphs are sinusoidal, but all the graphs start at different points on the sine curve. Looking at graphs starting from equilibrium, the difference between displacement and velocity is a quarter of the time period or T/4. Similarly, the difference between displacement and acceleration is T/2

Period T - 360° or 2π rads T/2 = 180° or π rads T/4 = 90° or $\pi/2$ rads	Useful equation for Phase difference: **Phase diff = (Shift/Period) x 360°** Change to radians as appropriate

Phase diff can also be calculated using the equation **$y = A \sin(\omega t - 2\pi x/\lambda)$**

9.1 Simple Harmonic Motion

SHM graphs (Displacement-Time, Velocity-Time, Acceleration-Time)

y-axis	Starting from equilibrium at t=0	Starting from a maximum point at t=0
Displacement	 $x = x_0 \sin \omega t$	 $x = x_0 \cos \omega t$
Velocity (1st derivative of displacement)	 $v = \omega x_0 \cos \omega t$	 $v = -\omega x_0 \sin \omega t$
Acceleration (2nd derivative of displacement)	 $a = -\omega^2 x_0 \sin \omega t$	 $a = -\omega^2 x_0 \cos \omega t$

Equations of Displacement, Velocity, and Acceleration

$x = x_0 \sin \omega t$

Velocity is the 1st derivative of displacement, so $v = \omega x_0 \cos \omega t$

Acceleration is the 2nd derivative of displacement, so $a = -\omega^2 x_0 \sin \omega t$

$a = -\omega^2 x_0 \sin \omega t = -\omega^2 (x_0 \sin \omega t)$

$x = x_0 \sin \omega t$, so $\mathbf{a = -\omega^2 x}$

This equation, $a = -\omega^2 x$, fits the definition of SHM: **motion in which the acceleration is proportional to the displacement from a fixed point and is always directed towards that fixed point**

When ω is constant, it is equal to θ/t. One complete revolution has an angle of 2π, and the time taken to cover this is T. Therefore,

$$\boxed{\omega = \frac{2\pi}{T}} \quad \boxed{\omega = 2\pi f}$$

The velocity equation

From earlier, $v = \omega x_0 \cos \omega t$, and $x = x_0 \sin \omega t$

We start with a trigonometric identity:

$\sin^2\Theta + \cos^2\Theta = 1$

$\cos \Theta = \pm\sqrt{(1 - \sin^2\Theta)}$

$$\boxed{v = \pm\omega\sqrt{{x_0}^2 - x^2}}$$

$\omega = \Theta/t$, so $\Theta = \omega t$

$\cos \omega t = \pm\sqrt{(1 - \sin^2 \omega t)}$, Therefore, $v = \pm\omega x_0\sqrt{(1 - \sin^2 \omega t)}$

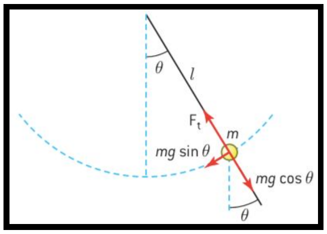
$x = x_0 \sin \omega t$, so $\sin \omega t = x/x_0$, and $\sin^2 \omega t = (x/x_0)^2$

Therefore, $v = \pm\omega x_0\sqrt{(1 - (x/x_0)^2)} = \pm\omega\sqrt{({x_0}^2 - {x_0}^2((x/x_0)^2)}$ {Took the x_0 inside the bracket by squaring}

So, $v = \pm\omega\sqrt{({x_0}^2 - x^2)}$

SHM Systems

Simple Pendulum



The simple pendulum represents a straightforward system that oscillates with SHM when its amplitude is small. The diagram shows the forces acting on the pendulum bob. The bob is in equilibrium along the radius of the string when the force F_t equals the component of weight in line with the string (mg cos Θ). The component of weight normal to the string provides the restoring force (mg sin Θ).

The time period of a simple pendulum

As per Newton's second law, the restoring force must equal the mass multiplied by acceleration:

mg sin Θ = ma. Since pendulums only work for small angles, and **sin Θ = Θ at small angles, Θ = x/l** (arc length from math)

So **-m (g/l) x = ma** (minus sign is because displacement is in the opposite direction to acceleration)

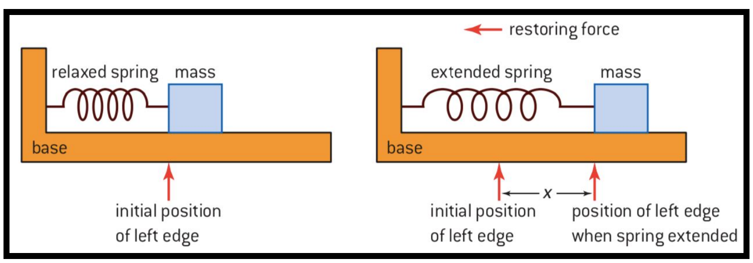
Cancelling m, **-(g/l) x = a**

Comparing to **a = -ω²x, ω² = (g/l), ω = √(g/l)**

T = 2π/ω, so **T = 2π√(l/g)**

$$\boxed{T = 2\pi\sqrt{\frac{l}{g}}}$$

Mass-spring system



We assume that the friction between the mass and the base is negligible. The mass exchanges elastic PE (when it is fully extended and compressed) with KE (as it passes through the equilibrium

The time period for a mass-spring system

The restoring force in this case will be F = -kx

Using Newton's 2nd Law,

ma = -kx

a = $-\omega^2$x, so -mω^2x = -kx

mω^2 = k

ω^2 = k/m

$(2\pi/T)^2$ = k/m

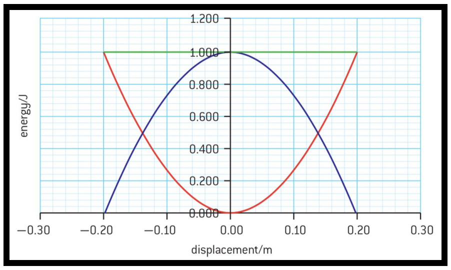
$(4\pi^2/T^2)$ = k/m

$T^2 = (4\pi^2 m)/k$

T = 2π √(m/k)

$$\boxed{T = 2\pi\sqrt{\frac{m}{k}}}$$

Energy in SHM systems



In a pendulum, there is an energy interchange between KE and GPE. In a mass-spring system, the same is between KE and Elastic PE. Knowing that $E_k = 0.5mv^2$, where $v = \pm\omega\sqrt{(x_o^2 - x^2)}$:

$\mathbf{E_k = 0.5m\omega^2(x_o^2 - x^2)}$

So $E_{kmax} = \mathbf{E_T = 0.5m\omega^2(x_o^2)}$ (if KE is at max, PE must equal 0)

So $E_P = E_T - E_K = 0.5m\omega^2(x_o^2) - 0.5m\omega^2(x_o^2 - x^2) = \mathbf{0.5m\omega^2x^2}$

The graph shows the variation of E_T (green), E_K (blue), and E_P (red):

Earlier we looked at the variation of energy with displacement. We can also look at the variation of energy with time. Once again we start with $E_K = 0.5mv^2$), where $v = \omega x_o \cos \omega t$:

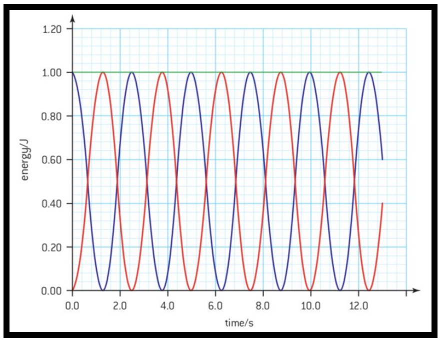
$E_K = 0.5m(\omega x_o \cos \omega t)^2 = \mathbf{0.5m\omega^2x_o^2\cos^2\omega t}$

When 'cos ωt' equals one, this gives the maximum kinetic energy, which is numerically equal to the total energy (because when KE is max, PE is 0, so TE = KE_{MAX})

Therefore, $E_{kmax} = E_T = \mathbf{0.5m\omega^2x_o^2}$

$E_P = E_T - E_K = 0.5m\omega^2x_o^2 - 0.5m\omega^2x_o^2\cos^2\omega t = 0.5m\omega^2x_o^2\ (1 - \cos^2\omega t) = \mathbf{0.5m\omega^2x_o^2\sin^2\omega t}$

These can also be represented graphically. TE is shown by the green line, PE by red, and KE by blue:

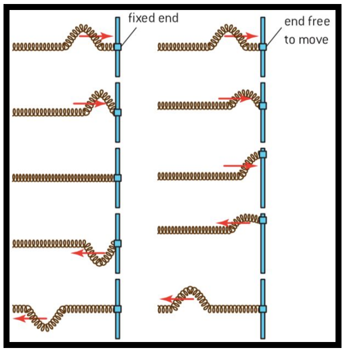


4.2 - Traveling Waves

Waves are of two fundamental types:

- **Mechanical waves** - which require a medium of travel
- **Electromagnetic waves** - which can travel through a vacuum

Depending on the nature of the endpoint, the reflection of a traveling wave can vary. This can be demonstrated using a slinky:



If the end is fixed, the pulse undergoes a phase change upon reflection. What was once an upward pulse is now a downward pulse. The phase change is π.

When the end is free to move, reflection occurs, but there is zero phase change, so an upward pulse returns as an upward pulse

There are two types of traveling waves:

- **Transverse waves** are waves for which the oscillation of particles is perpendicular to the direction of energy transfer
- **Longitudinal waves** are waves for which the oscillation of particles is parallel to the direction of the energy transfer

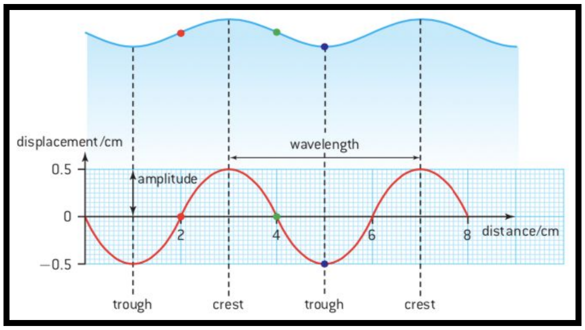
Describing waves

- **Wavelength (λ)** - shortest distance between two points that are in phase. Measured in **meters (m)**.
- **Frequency (*f*)** - number of waves passing a fixed point in one second. Measured in **Hertz (Hz).**
- **Period (T)** - time taken for one complete wavelength to pass a fixed point. Measured in **seconds (s)**.
- **Amplitude (A)** - maximum displacement of a wave from its rest position. Measured in **meters (m)**

Displacement-distance graphs

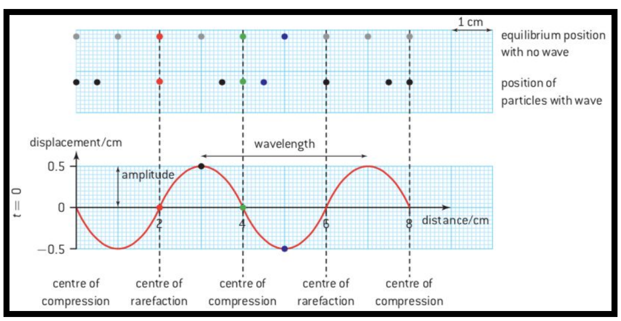
1) **Transverse waves**

For transverse waves, the distance and displacement are perpendicular.

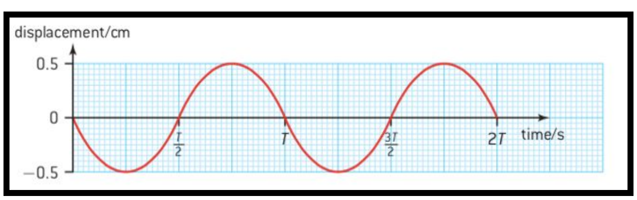


2) **Longitudinal waves**

For longitudinal waves, the distance and displacement of the wave are parallel. Therefore, the displacement-distance graph of longitudinal waves allows us to identify **compressions** (areas of high pressure) and **rarefactions** (areas of low pressure)



Displacement-time graph



From this graph, it is easy to find out the time period and the amplitude of the wave

The wave equation

The velocity of a wave is given by the equation: $v = f\lambda$

From the displacement-distance and the distance-time graphs, you can determine T and λ. Using $v = d/t$, $v = \lambda/T$. Since $T = 1/f$, $v = f\lambda$.

Electromagnetic waves

WAVE	**PROPERTIES**	**Examples, Uses, and Effects**
RADIO WAVES		Radios, TV broadcasts, NMR spectroscopy
MICROWAVES		Heating food in microwave ovens, satellites, telephones. Can heat body cells
INFRARED RADIATION		Radiant heaters and grills, TV remotes, security alarms, lamps. Emits heat
VISIBLE LIGHT (400-700nm)		Human sight
ULTRAVIOLET (UV RAYS)		Causes tanning, skin cancer, and eye damage. Kills bacteria, Fluorescence. Sterilizing food and medical instruments
X-RAYS		Used for X-ray photography, causes cancer but can also be used for cancer treatment
GAMMA RAYS		Emitted by radioactive materials, uses and effects as for X-rays, used for sterilizing medical equipment and food

Electromagnetic waves share similar features:

- They can travel through a vacuum
- Travel at a speed of 3.0×10^8 m/s
- They are transverse waves
- They transfer energy

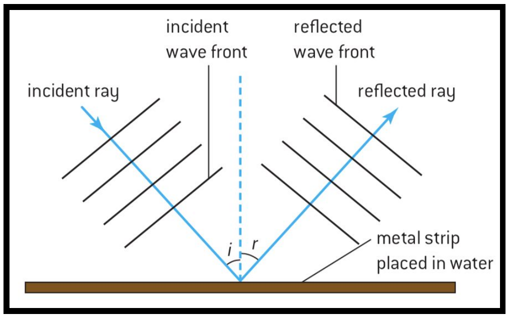
4.3 - Wave characteristics

Wavefronts vs rays

- A **wavefront** is a line joining the points that are in the same phase
- A **ray** is a line that indicates the direction of propagation of the wave
- **Rays are perpendicular to wavefronts**

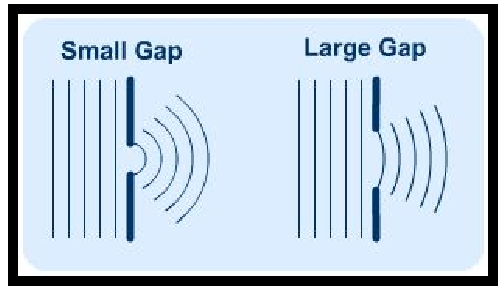
Wavefront and ray diagrams

Reflection



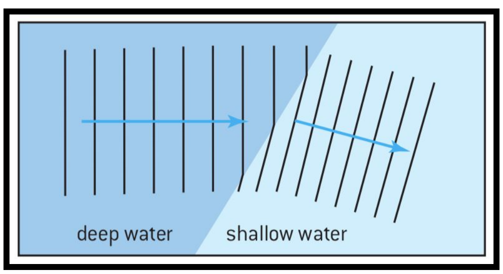
There is no change in the wavelength, and the angle of incidence (i) is equal to the angle of reflection (r)

Diffraction



The bending of waves around an obstacle or, the spreading out as they pass through a gap is known as diffraction. Diffraction is only significant if the size of the gap is about the same as the wavelength. Wider gaps produce less diffraction

Refraction

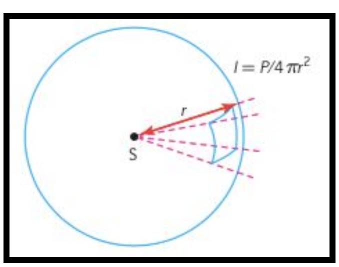


Different parts of the wave hit the boundary before others, which causes the wave to change direction

As the wavefronts enter the denser medium, they slow down, and the wavelength decreases while frequency remains constant ($v = f\lambda$).

Wave intensity

The loudness of a sound, or the brightness of a light depends on the energy that is being received by the observer. Energy is found to be proportional to the square of the amplitude. If we picture waves being emitted from a point source, waves move in all directions. The diagram below illustrates this:



Since the energy transferred per second is **power**, the intensity at a point that is 'r' m away from the source can be written as:

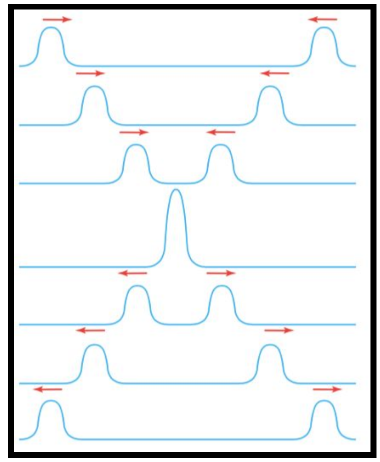
$$\boxed{I = \frac{P}{4\pi r^2}}$$

This equation shows that intensity has an **inverse-square relationship** with the distance from the point source, which can be written as: $\mathbf{I \propto x^{-2}}$. $P = E/t$, $E_T = 0.5kx_0^2$. As $P \propto E_T$, $P \propto A^2$, so $\mathbf{I \propto A^2}$

Intensity is therefore the power being transmitted per unit area. It is measured in $\mathbf{Wm^{-2}}$

Superposition

The **principle of superposition** states that when 2 or more waves meet, the resultant displacement is equal to the sum of the individual displacements.



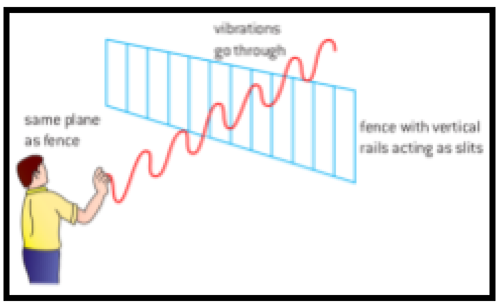
Constructive interference occurs when both waves have the same sign displacement.

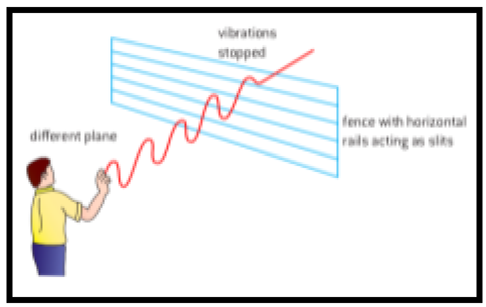
Destructive interference occurs when one wave has +ve displacement and the other has -ve displacement.

Polarization

Although transverse and longitudinal waves have common properties - they reflect, refract, diffract, and superpose - the difference between them can be seen by the property of polarization.

Polarization of **transverse waves** restricts the direction of oscillation to a plane perpendicular to the direction of propagation:

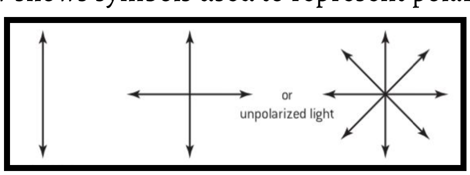




Longitudinal waves do not exhibit polarization because, for these waves, the direction of oscillation is parallel to the direction of propagation.

Polarized light is light for which the electric field vector vibrates in one plane. Most naturally occurring EM waves are completely **unpolarized**. This means that the electric field vectors vibrate in **random directions**.

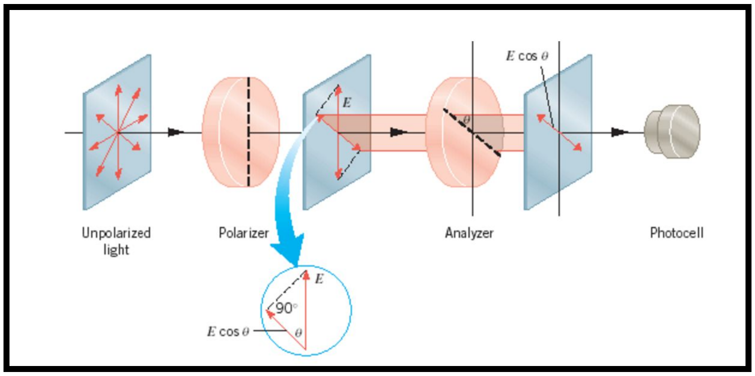
When the direction of vibration stays constant over time, the wave is said to be **plane-polarized** in the direction of vibration. **Partial polarization** is when there is some restriction to the direction of vibration but not 100%. The diagram below shows symbols used to represent polarized and unpolarized light.



The most common way to produce polarized light in the modern day is using **Polaroids**. **Polarizers** and **Analyzers** (both polaroids) can be used to do so.

Malus' Law

The **transmission axis** is the direction of polarization that a polarizer allows through



The **polarizer** allows **half** the original intensity through since half of the components of all waves are parallel to the transmission axis. **Analyzers** are used to detect polarised light. 100% light must pass through to ensure polarization. When the transmission axis of the analyzer is:

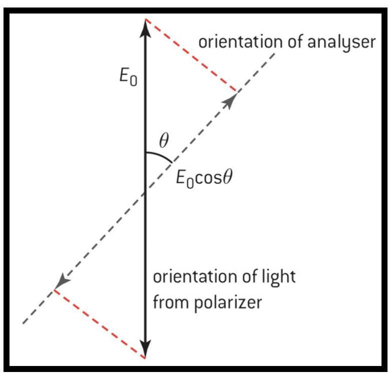
- Parallel to the polarized light, Intensity is maximum
- Perpendicular to polarized light, Intensity is minimum

When the analyzer is neither perpendicular nor parallel, we use Malus' Law which states that:

$$\boxed{I = I_{MAX} \cos^2 \theta}$$

Where **0° ≤ θ ≤ 90°**

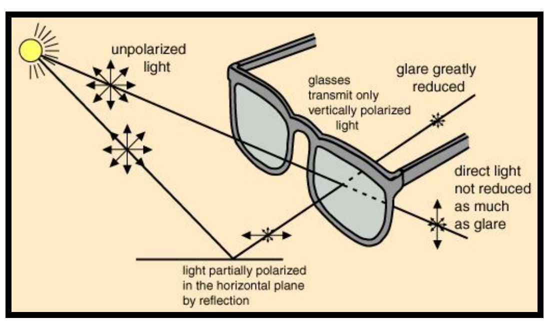
***θ* is the angle between the transmission axis of the analyzer and the polarized light**



Uses of analyzers

Stress analyzers - when metals are placed between a polarizer and an analyzer, the regions of highest stress appear as white light.

Working of a pair **polaroid glasses** on a reflected image:



Light is **horizontally** polarized by reflection on the surface. The sunglasses have a transmission axis that is perpendicular to the reflected ray. This means that the intensity of light is reduced. Therefore, the glare on objects is reduced while wearing polaroid glasses

4.4 - Wave behavior

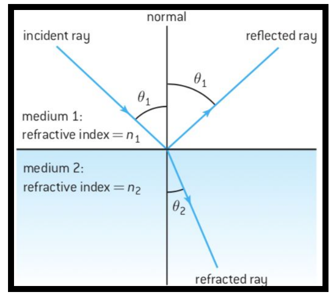
Reflection and Refraction

The laws of reflection and refraction can be summarised as follows:

1) The reflected and refracted rays are on the same plane as the incident ray and normal
2) The angle of incidence equals the angle of reflection
3) For waves of a particular frequency and for a chosen pair of media the ratio of the sine of the angle of incidence to the sine of the angle of refraction is the refractive index (**Snell's Law**):

$$\boxed{\frac{n_1}{n_2} = \frac{\sin\theta_2}{\sin\theta_1} = \frac{v_2}{v_1}}$$

The diagram on the right shows reflection and refraction occurring. Ray diagrams as such do not show what is happening to the wavelengths of the waves.



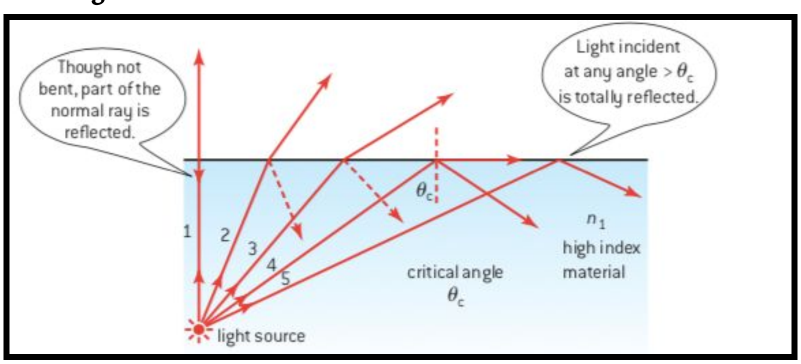
Refractive index and Snell's Law

The **absolute refractive index (n)** of a medium is defined in terms of the speed of EM waves as:

$$\boxed{n = \frac{\text{speed of electromagnetic waves in a vacuum}}{\text{speed of electromagnetic waves in the medium}} = \frac{c}{v}}$$

The critical angle and Total Internal Reflection (TIR)

When light passes from a **more optically dense medium to a less optically dense medium**, it speeds up and bends away from the normal. Increasing the angle of incidence on this boundary results in a greater angle of refraction. **The angle of incidence in the denser medium for which the angle of refraction is 90° is known as the critical angle.**



When the angle of incidence > critical angle, all light is TOTALLY INTERNALLY REFLECTED

Calculating the critical angle

Snell's Law gives: $\boxed{\frac{n_1}{n_2} = \frac{\sin\theta_2}{\sin\theta_1} = \frac{v_2}{v_1}}$

To obtain a critical angle, medium 1 must be more optically dense than medium 2. When $\Theta_1 = \Theta_c$ then $\Theta_2 = 90°$, so $\sin \Theta_2 = 1$.

This gives :

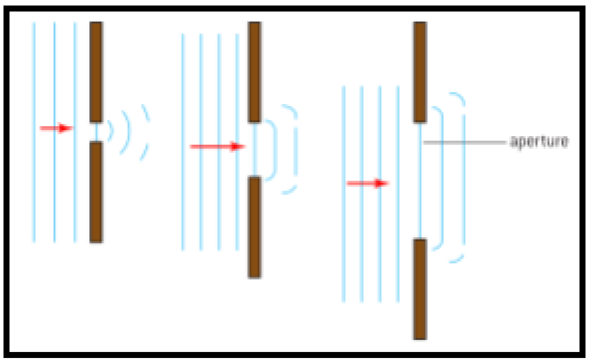
$$\boxed{\sin \theta_c = {}_1n_2 = \frac{n_2}{n_1}}$$

When the less dense medium is vacuum or air, $n_2 = 1$. So:

$$\boxed{\sin \theta_c = \frac{1}{n_1}}$$

<u>Diffraction</u>

Diffraction is the spreading of a wave when passing around an aperture (a gap) or an obstacle.

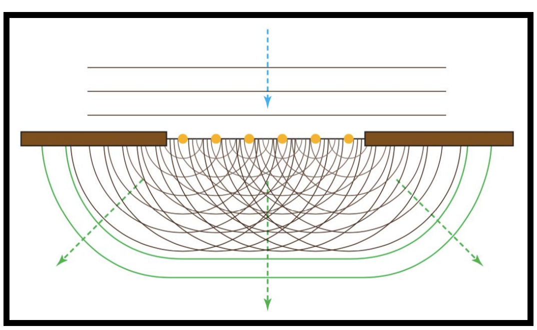


Observations of diffraction patterns include:

- The frequency, wavelength, and speed of the waves remains the same after diffraction
- The direction of propagation and the pattern of the waves change
- The effect of the diffraction is most obvious when the aperture width is approximately the same as the wavelength of the waves
- The amplitude of the diffracted wave is less than that of the incident wave because the energy is distributed over a larger area

<u>Huygens-Fresnel principle</u>

This principle gives a good insight into how the single-slit diffraction pattern comes about:

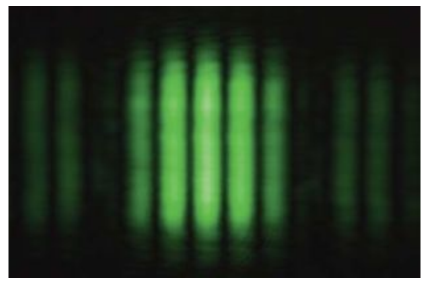


Plane waves traveling towards the slit behave as if they were sources of secondary wavelets. The orange dots show these ‘secondary sources’ within the slit. These ‘sources’ each spread out as circular waves. The tangents to these waves will now become the new wavefront.

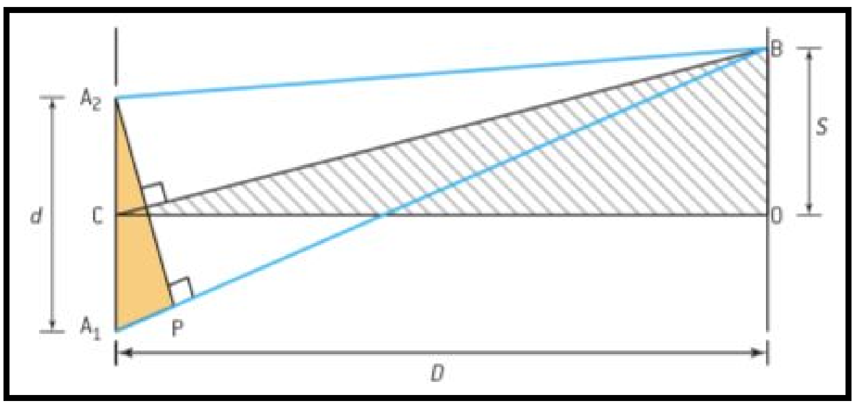
Double-slit interference

When two or more waves combine to produce a new wave, **interference** has occurred. When the resultant wave has a larger amplitude, than the individual waves, the interference is said to be **constructive**. When the resultant has a smaller amplitude, the interference is **destructive**. Interference can be achieved by using two similar sources of all types of waves. It is only observable if the two sources have a **constant phase relationship**. To be in this relationship the two sources must be **coherent**. **Coherent sources** are mandatory for observable **interference** patterns because the rate of change of phase at a given point is constant for both the **sources** and hence minima and maxima i.e. constructive and destructive **interference** can be seen simultaneously at different points on the screen

When two diffracted beams cross, interference occurs. A pattern of equally spaced bright and dark fringes is obtained on a screen positioned in a region where the beams overlap. When a crest meets a crest (or trough meets trough), constructive interference occurs. When a crest meets a trough, destructive interference occurs.



Path difference and the double-slit equation



The diagram shows two apertures A_1 and A_2 distance d apart. The double-slit is distance D from a screen. O is the position of the central bright fringe and B is the next bright fringe above O. The distance OB is the fringe spacing s. Angle A_1A_2P is equal to angle BCO, as the yellow and shaded triangle can be considered to be similar because of how small the angles are. Let A_1A_2P = BCO = Θ.

$\tan \Theta = s/D$

$\Theta = s/D$ {$\tan \Theta = \Theta$ due to small angle approx}

$\sin \Theta = \lambda/d$

$\Theta = \lambda/d$

$$\boxed{s = \frac{\lambda D}{d}}$$

$s/D = \lambda/d$

$s = \lambda D/d$

In general, for two coherent beams starting in phase:

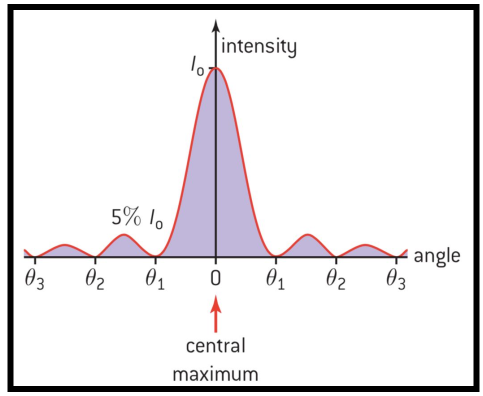
Constructive interference occurs If the path difference is a whole number of wavelengths
Path difference = nλ {n = 0,1,2…}
Destructive interference occurs if the path difference is an odd number of half wavelengths
Path difference = (n+0.5)λ {n = 0,1,2…}

'n' is the order of the fringe

9.2 - Single-slit diffraction

Graph of intensity against angle

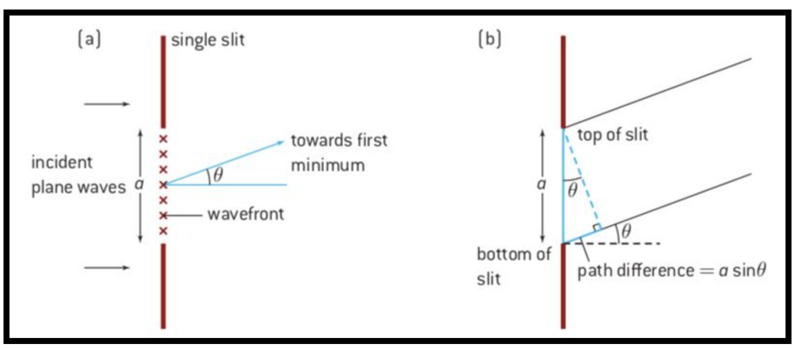
- Central maximum is twice as wide as the other fringes
- Central maximum is at least 5 times brighter than the other fringes
- The pattern becomes more spread out if the difference between the size of the wavelength and the slit width decreases



The single slit equation

The given equation gives the angle to the first minima from the central maximum:

$$\theta = \frac{\lambda}{a}$$

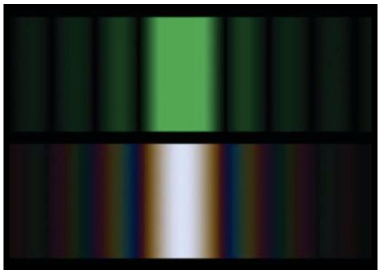


Waves coming in from the edges have a path difference of a sin Θ

Waves from the point halfway along the slit will have a path difference of 0.5a sin Θ.

When 0.5a sin Θ is equal to half a wavelength (Phase diff between central max and first min) sin Θ is equal to λ/a. Because we are dealing with small angles, we can approximate sin Θ to Θ. Therefore, we arrive at the equation Θ = λ/a. Greater wavelength gives a greater angle, Greater slit length gives a smaller angle

Single-slit with monochromatic and white light



The upper image is obtained from green light. The lower image is obtained from white light. Both the angular width of the central max and the angular separation of successive secondary maxima depend on the wavelength of the light. For the white light, for the second maxima, the violet light is less deviated than the other colors as it has the shortest λ

The edges of the principle maximum are colored rather than pure white. This is because the principal maxima for the colors at the blue end of the visible spectrum are less spread than the colors at the red end; the edges are therefore a combination of red, orange, and yellow.

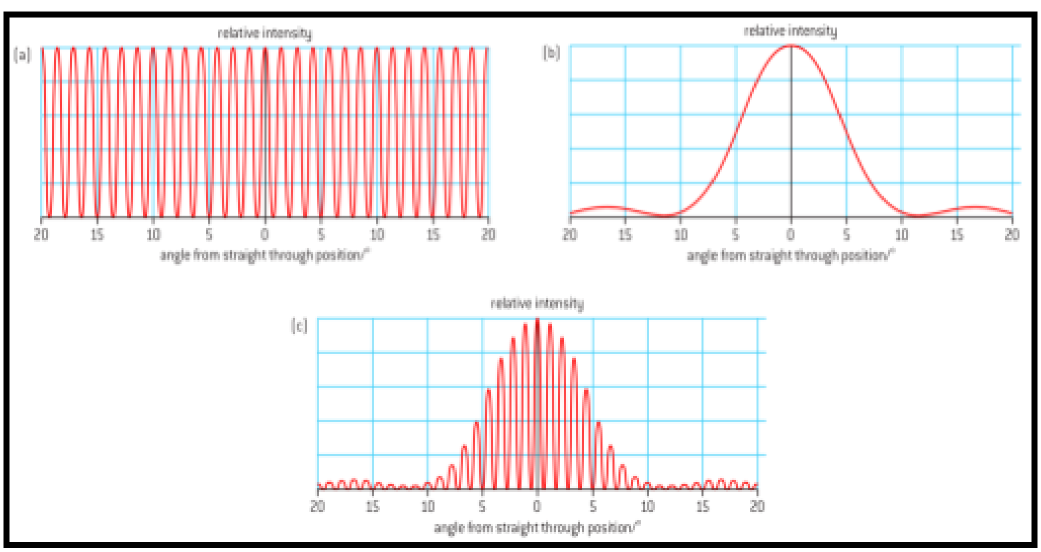
9.3 - Interference

For two sources of waves to interfere then these conditions must be met:

- Similar amplitude
- Same wavelength
- Constant phase difference

We know that a single slit produces a diffraction pattern with a very intense principal maximum and much less intense secondary maxima. A double-slit is two single slits, so each of the slits produces a diffraction pattern, and the waves from the two slits interfere.

Figure (a) shows how the relative intensity would vary for a double-slit pattern without any modification due to diffraction (ideal situation). By using relative intensity we avoid the need to think about the actual intensity values. Figure (b) shows the variation of the relative intensity with angle for a single slit. Figure (c) shows the superposition of the two effects so that the single-slit diffraction behaves as the envelope of the interference pattern. Shaping a pattern in this way is known as **modulation**.



We know from earlier that the fringe spacing is given by the equation:

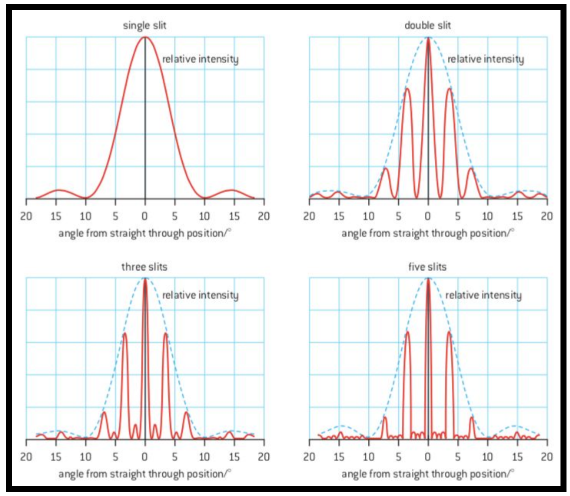
$$\boxed{s = \frac{\lambda D}{d}}$$

s = fringe width
D = distance between slits and screen
d = distance between slits

Multiple-slit interference

If there are more than two slits:

- The sharpness of the fringes increases (fringes are narrower)
- There are faint secondary maxima between primary maxima
  - The no. of secondary maxima is given by the equation (N-2), where N is the no.of slits
- The intensity of the central maximum is increased



Diffraction grating

Diffraction gratings are used to produce **optical spectra**. A grating contains a large number of parallel, equally spaced slits or "lines". Different wavelengths are diffracted at different angles, producing interference maxima at an angle Θ given by:

$$\mathbf{n\lambda = d \sin \Theta}$$

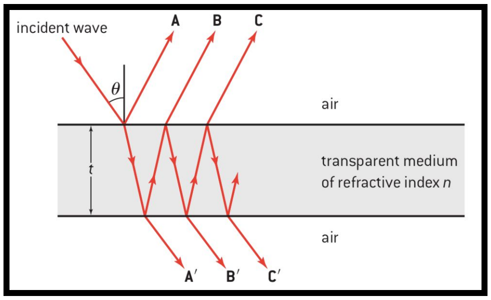
n is any +ve integer that represents the order of the maximum. n = 0 gives the angle at the central maximum (which is 0°). n = 1 gives the angle to the first maximum on either side of the central, and so. 'd' is the distance between the slits in the grate.

One use of diffraction grating is to disperse white light into its component colors. This is because different wavelengths of light in the visible spectrum produce maxima at different angles. Each successive visible spectrum repeats the order of the colors of the previous one but becomes less intense and more spread out. It is usual for the "number of lines per mm (N)" to be quoted for a diffraction grating. This needs to be converted to the distance between the lines. To do so, use the following equation: **d = 1/N.**

Thin-film interference

Thin-film interference is based on the following properties:

- When a wave hits the boundary going from a denser to less dense medium, it reflects IN PHASE
- When a wave hits the boundary going from a less dense to more dense medium, it reflects OUT OF PHASE
- In any case, the wave is always transmitted through the boundary IN PHASE with the original pulse



This diagram shows a wave incident at angle Θ to the surface of a film of transparent material (eg: oil) having a refractive index n. Θ and the thickness of the film, t, are very small, so the incident wave is effectively normal to the surface. The incident wave partially reflects at the top of the surface and partially refracts into the film. The refracted wave reaching the lower surface of the film is reflected and refracted partially. This can occur several times.

When A is reflected from the top surface of the film, there is a phase change of π rads (equivalent to half a λ), because the reflection is at an optically denser medium. Wave B travels an optical distance of 2tn before refracting back into the air. Thus the optical path difference between wave A and B is 2tn. If there had been no phase change then this optical distance would equal mλ for constructive interference. However, because of A's phase change at the top surface, the overall effect will be destructive interference.

Thus, for the light reflecting from the film when

$$\mathbf{2tn = m\lambda}$$

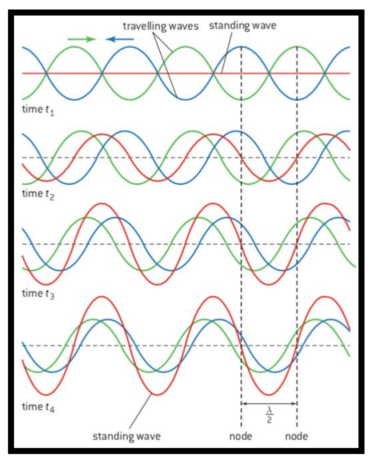
there will be destructive interference and when

$$\mathbf{2tn = (m + 0.5)\lambda}$$

there will be constructive interference

These equations flip when the air below the thin film is replaced with an even denser medium

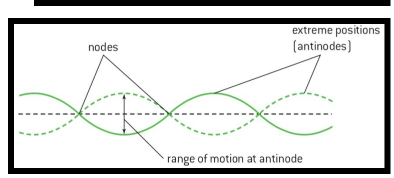
4.5 Standing Waves



Standing waves are formed due to the **reflection and superposition** of two identical waves moving in opposite directions

A **node** is a region of a point on a standing wave where displacement is 0. **The distance between two successive nodes is λ/2.** An **antinode** is a region or a point on a standing wave where displacement is maximum.

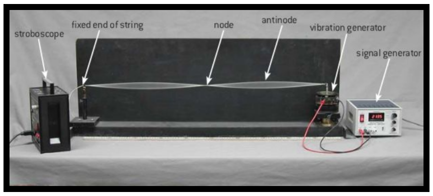
The diagram below shows two traveling waves (blue and green) moving towards each other at times t_1, t_2, t_3, t_4. The green and blue waves superpose to give the red standing wave.



Over a complete time period of an oscillation, the standing wave will occupy a variety of positions as shown on the right:

Melde's String

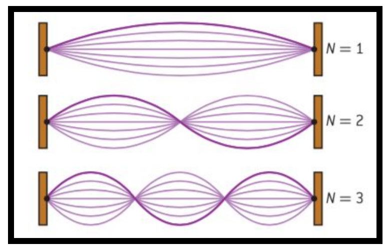
This apparatus, developed by Franz Melde, is useful in demonstrating standing waves on a string



A string is strung between a vibration generator and a fixed end. When the vibration generator is attached to an audio frequency generator, the end of the string attached to the vibration generator oscillates vertically

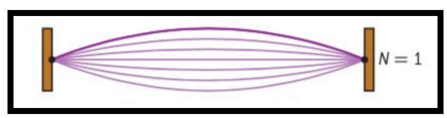
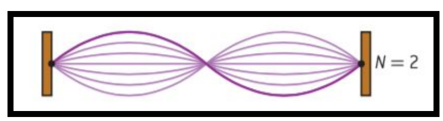
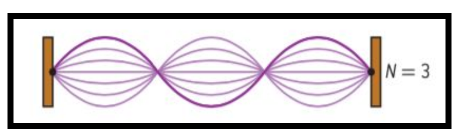
A wave travels down the string before undergoing a phase change of 180° when it reflects at the fixed end. This reflected wave superposes with the incident wave (at certain frequencies) to form a standing wave. The frequency is raised from zero until eventually a frequency is reached where the string vibrates with large amplitude in the form of a single loop - **the first harmonic**. If the frequency is increased further, the amplitude dies down until twice the frequency of the first harmonic is reached - in this case, we get two loops - **the second harmonic**.

Boundary conditions on a string



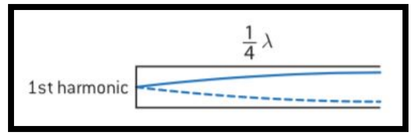
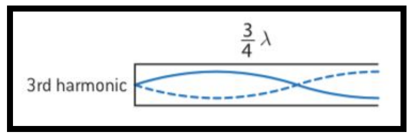
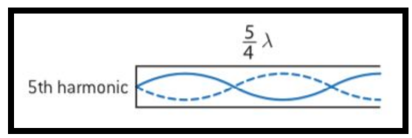
- Both ends of the string are fixed ends
- This means there are only nodes at the ends
  - Upon plucking the string, traveling waves move towards the fixed ends and are reflected with a phase difference of π. The reflected waves return to interfere with the incident waves at specific frequencies to form a standing wave

The frequencies of the string can be calculated:

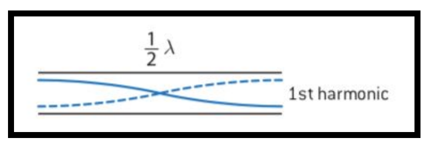
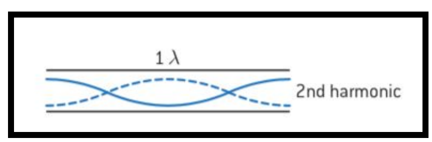
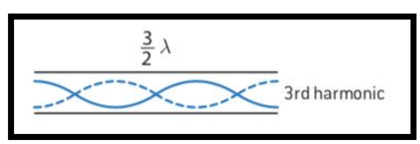
$f_1 = v/\lambda$ Length (L) of the string is equal to $\lambda/2$ (distance between nodes). Therefore, $\lambda = 2L$ The first harmonic frequency on a string is $\boldsymbol{f_1 = v/2L}$	
$f_2 = v/\lambda$ Length (L) of the string is equal to λ (2 x dist between nodes) Therefore, $\lambda = L$ The second harmonic frequency on a string is $\boldsymbol{f_1 = v/L}$	
$f_3 = v/\lambda$ Length (L) of the string is equal to $3\lambda/2$ (3 x dist between nodes), Therefore, $\lambda = 2L/3$ The third harmonic frequency on a string is $\boldsymbol{f_1 = 3v/2L}$	
Similarly, equations for 4th harmonic, 5th harmonic, and so on can be calculated **String series have odd and even harmonics**	

Boundary conditions for a closed pipe (one end closed, one open)

In pipes, longitudinal waves are created (rather than transverse waves). Sound waves are reflected at both ends of a pipe, irrespective of whether they are open or closed. There is an **antinode at open ends** (no phase change upon reflection, so the incident wave isn't canceled) and there are **nodes at closed ends** (fixed end causes phase change upon reflection, so the incident wave is canceled out).

$f_1 = v/\lambda$ Length (L) of the pipe is equal to $\lambda/4$ (½ dist between nodes) Therefore, $\lambda = 4L$ Therefore, the first harmonic frequency on a string is $\boldsymbol{f_1 = v/4L}$	
$f_1 = v/\lambda$ Length (L) of the pipe is equal to $3\lambda/4$ (1.5 x dist between nodes) Therefore, $\lambda = 4L/3$ Therefore, the third harmonic frequency on a string is $\boldsymbol{f_1 = 3v/4L}$	
$f_1 = v/\lambda$ Length (L) of the pipe is equal to $5\lambda/4$ (2.5 x dist between nodes) Therefore, $\lambda = 4L/5$ Therefore, the fifth harmonic frequency on a string is $\boldsymbol{f_1 = 5v/4L}$	
In one side closed tubes, even harmonics are missing	

Boundary conditions for an open pipe (both ends open)

$f_1 = v/\lambda$ Length (L) of the pipe is equal to $\lambda/2$ (distinguish between nodes) Therefore, $\lambda = 2L$ Therefore, the first harmonic frequency on a string is $\boldsymbol{f_1 = v/2L}$	
$f_1 = v/\lambda$ Length (L) of the pipe is equal to λ (2 x dist between nodes) Therefore, $\lambda = L$ Therefore, the first harmonic frequency on a string is $\boldsymbol{f_1 = v/L}$	
$f_1 = v/\lambda$ Length (L) of the pipe is equal to $3\lambda/2$ (3 x dist between nodes) Therefore, $\lambda = 2L/3$ Therefore, the first harmonic frequency on a string is $\boldsymbol{f_1 = 3v/2L}$	
String series and open pipes have both odd and even harmonics	

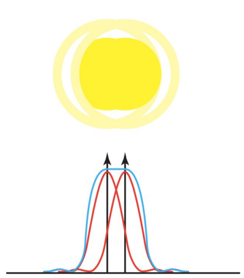
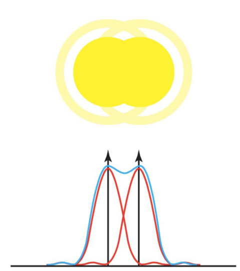
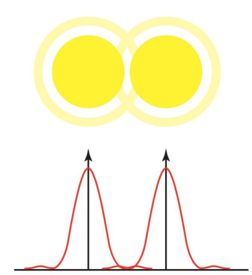
Comparison of traveling waves and stationary waves

Property	Traveling-wave	Standing wave
Energy transfer	Energy is transferred in the direction of propagation	No energy is transferred by the wave although there is an interchange of kinetic and potential energy within the standing wave
Amplitude	All particles have the same amplitude	Amplitude varies within a loop - maximum occurs at an antinode and zero at a node
Phase	Within a wavelength, the phase is different for each particle	Particles within a "loop" are in phase and antiphase [180° out of phase] with the particles in adjacent "loops"
Wavelength	The distance between adjacent particles which are in phase	Twice the distance between adjacent nodes (or adjacent antinodes)
Frequency	All particles vibrate with the same frequency	All particles vibrate with the same frequency except at nodes (which are stationary)

9.4 - Resolution

Resolution is the ability of an imaging system to be able to produce two distinguishable images of two separate objects. When there are two sources of light, two diffraction patterns will be formed by the system. Two objects observed through an aperture produce two diffracted images that may overlap.

The **Rayleigh criterion** states that two sources are said to be **just resolved** if the first min of the diffraction pattern of one source falls on the central max of the diffraction pattern of another source

 images unresolved	 images just resolved	 images fully resolved
The principal maximum of one diffraction pattern lies closer to the second pattern than its first minimum	The principal maximum of one diffraction pattern is at the same position as the first minimum of the second pattern	The principal maximum of each diffraction pattern lies further from the other than the principal maximum

Resolution equation

For single slits, we saw that the first minimum occurs when the angle with the straight-through position is given by $\Theta = \lambda/b$. With a circular aperture, the equation is modified with a factor of 1.22. So:

$$\boxed{\theta = 1.22\,\frac{\lambda}{a}}$$

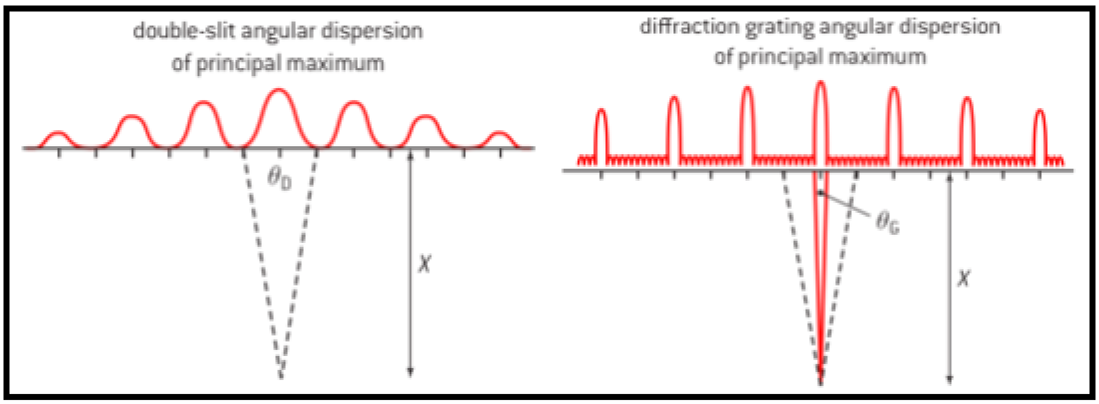
a here is the diameter of the aperture.

Ways in which to increase the resolution of a device:

- Increase the diameter
- Decrease the wavelength
- Increase the frequency (which decreases the wavelength)

Resolvance of diffraction gratings

From diffraction grating, we know that increasing the number of slits improves the sharpness of the maxima formed. The diagrams show that when the light of the same wavelength is viewed from the same distance x, the angular dispersion Θ_D for the principal maximum with the double-slit is larger than the angular dispersion Θ_G for the grating. A sharper principal maximum is one with less angular dispersion. With wider maxima, there is more overlap of images from different sources and lower resolution.



Using this argument, we see that when light is incident on a grating, a wider beam (which covers more lines) will produce sharper images and better resolution.

The resolvance (R) for a diffraction grating is defined as the ratio of the average wavelength (λ) of 2 wavelengths to the difference in wavelengths (λ/2). The resolvance is also equal to 'Nm', where 'N' is the total number of slits illuminated by the incident beam and 'm' is the order of the diffraction

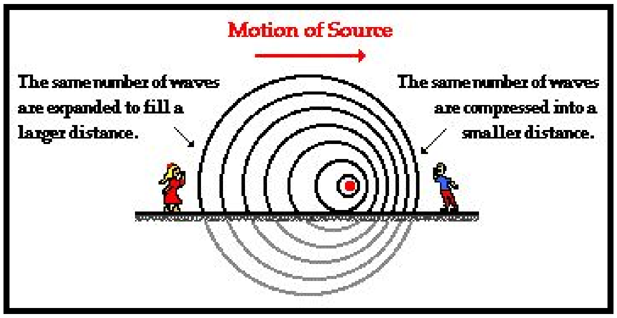
$$\boxed{R = \frac{\lambda}{\Delta\lambda} = Nm}$$

9.5 - Doppler effect

Doppler effect is the apparent change in the frequency of a wave motion when there is relative motion between the source of the wave and the observer. Doppler effect can occur for both light and sound.

Doppler effect for moving sources

When the source of the light or sound is moving, and the observer is stationary, the **speed of the source wave is always constant**



Source moves towards a stationary observer	Source moves away from a stationary observer
$\lambda' = \lambda - s$ $\lambda' = \lambda - u_s t$ {$s = u_s t$} {u_s is the speed of the source} $\lambda' = \lambda - (u_s/f)$ {$t = 1/f$} $\lambda' = (v/f) - (u_s/f)$ {$v = f\lambda$} $\lambda' = (v-u_s)/f$ $v/f' = (v-u_s)/f$ {$v = f\lambda$} $f' = vf/(v-u_s)$ $\boxed{\lambda' = \frac{v - u_s}{f}}$ $\boxed{f' = \frac{v}{\lambda'} = f\left(\frac{v}{v - u_s}\right)}$	$\lambda' = \lambda + s$ $\lambda' = \lambda + u_s t$ {$s = u_s t$} {u_s is the speed of the source} Similar steps to previous case lead to: $\lambda' = (v+u_s)/f$ $v/f' = (v+u_s)/f$ $f' = vf/(v+u_s)$ $\boxed{\lambda' = \frac{v + u_s}{f}}$ $\boxed{f' = \frac{v}{\lambda'} = f\left(\frac{v}{v + u_s}\right)}$

Combining the equations in cases 1 and 2, we get the data booklet equation:

$$\boxed{f' = f\left(\frac{v}{v \pm u_s}\right)}$$

f' = apparent frequency
f = frequency of the source
v = wave speed in a vacuum (sound or light)
u_s = velocity of the source

Doppler effect for moving observer

When the observer is moving, and the source of light or sound is stationary, the **wavelength of the source wave is always constant**

Observer moving towards a stationary source	**Observer moving away from a stationary source**
The velocity of the sound relative to the observer is: $v + u_o$ $f' = v/\lambda$ $f' = (v + u_o)/\lambda$ $\lambda = (v + u_o)/f'$ $v/f = (v + u_o)/f'$ $f' = f(v + u_o)/v$ $$\boxed{f' = f\left(\frac{v + u_o}{v}\right)}$$	The velocity of the sound relative to the observer is: $v - u_o$ $f' = v/\lambda$ $f' = (v - u_o)/\lambda$ $\lambda = (v - u_o)/f'$ $v/f = (v - u_o)/f'$ $f' = f(v - u_o)/v$ $$\boxed{f' = f\left(\frac{v - u_o}{v}\right)}$$

Combining the equations in cases 1 and 2, we get the data booklet equation:

$$\boxed{f' = f\left(\frac{v \pm u_o}{v}\right)}$$

f' = apparent frequency
f = frequency of the source
v = wave speed in a vacuum (sound or light)
u_s = velocity of the observer

Doppler shift (Case for source moving towards the observer)

The apparent change in the frequency experienced as a result of the doppler effect is known as the **doppler shift**. This value increases as the relative velocity between the source and the observer increases.

$\Delta f = f' - f$
$\Delta f = \{vf/(v-u_s)\} - f$
$\Delta f = f[\{v/(v-u_s)\} - 1]$
$\Delta f/f = \{v/(v-u_s)\} - 1$
$\boldsymbol{\Delta f/f = u_s/(v - u_s)}$

$$\boxed{\frac{\Delta f}{f} \approx \frac{v}{c}}$$

If u_s is far less than c, then $\boldsymbol{\Delta f/f = u_s/v}$ {as $v - u_s \approx v$}

Furthermore, $\boxed{\dfrac{\Delta f}{f} = \dfrac{\Delta\lambda}{\lambda} \approx \dfrac{v}{c}}$ (Don't need to know why $\Delta f/f = \Delta\lambda/\lambda$)

Applications of Doppler effect

- Medicinal
  - Ultrasonic waves reflected from red blood cells are used to determine the velocity of blood flow
  - Reflection of ultrasonic waves can be used to detect the movement of the chest of a young fetus and to monitor the heartbeat
- Radar speed traps
  - Police monitor the speeds of vehicles with a radar gun