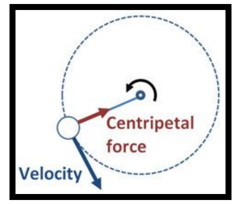
6.1 Circular Motion

A particle is in a **uniform circular motion** if it moves in a circular path with constant speed 'v'. **Linear velocity** is always **tangential** to the circular path. Even in a uniform circular motion, the body accelerates, as the direction of velocity constantly changes. This acceleration, and therefore, force, is **towards the center of the circle**.



The angle moved by an object from its starting position is the **angular displacement**. **Angular speed (ω)** is angular displacement over time.

$$\omega = \frac{\theta}{t}$$

Period and frequency

The time taken for an object to round the circle once is known as the **period (T)**. In one period, the distance traveled is 2π. So T, in rads s^{-1}, must be:

$$T = \frac{2\pi}{\omega}$$

Frequency(f) is the number of times an object goes around in a circle in unit time. T and f are linked by the equation:

$$T = \frac{1}{f}$$

This can be rewritten as

$$\omega = 2\pi f$$

(As $T = 2\pi/\omega$)

Angular and linear speed

When the circle has radius r, the distance circumference is $2\pi r$, and the time to cover this is T. So:

$v_{linear} = 2\pi r/T$

This can be rearranged to give: $T = 2\pi r/v$

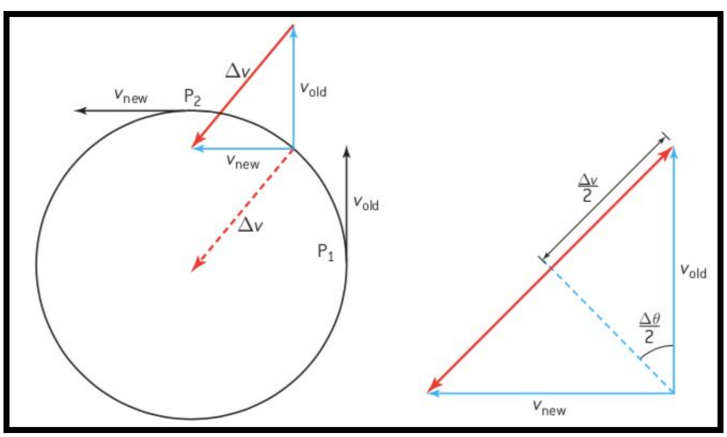
$T = 2\pi/\omega$, so, therefore,

$2\pi r/v = 2\pi/\omega$

After simplification: $v = \omega r$

$$v = \omega r$$

Centripetal acceleration



The diagram shows the velocity of a body at 2 different points in its circular path. Acceleration is the change in velocity over time. The change in velocity here is the resultant of the subtraction of the two vectors, which proves that the direction of acceleration is towards the center of the circle.

The time taken to move from P_1 to P_2 is given by:

$$\Delta t = \frac{r\Delta\theta}{v}$$

Using trig,

$$\frac{\Delta v}{2} = v \sin\left(\frac{\Delta\theta}{2}\right)$$

The acceleration towards the center of the circle can be written as:

$$a = \frac{\Delta v}{\Delta t} = \frac{2v \sin\left(\frac{\Delta\theta}{2}\right)}{\frac{2r}{v}\frac{\Delta\theta}{2}} \rightarrow a = \frac{v^2}{r}\frac{\sin\left(\frac{\Delta\theta}{2}\right)}{\frac{\Delta\theta}{2}}$$

When Θ is very small, the ratio sin (ΔΘ/2) / ΔΘ/2 is very close to 1, so the instantaneous acceleration between P_1 and P_2 is given by:

$$a = \frac{v^2}{r} = \omega^2 r = v\omega$$

Centripetal Force

F = ma, so:

$$\text{force} = m\frac{v^2}{r} = m\omega^2 r = mv\omega$$

Banking

A car can turn (bank) because of friction between the tire and the pavement. This friction points to the center of the circle. Some other forces that provide centripetal force are:

- Tension
- Friction
- Gravitational
- Electrical
- Magnetic

Horizontal circular motion

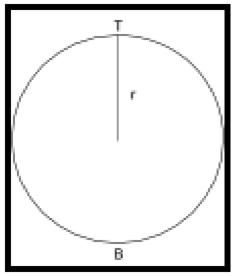
Centripetal force is equal to mv^2r^{-1} at all points on the path

Vertical circular motion

Tension at the top:	**Tension at the bottom:**
F_C at the top = Tension + Weight $F_C = T + W$ $T = F_C - W$ $T_{top} = mv^2r^{-1} - mg$	F_C at the bottom = Tension - Weight $F_C = T - W$ $T = F_C + W$ {At the bottom} $T_{bottom} = mv^2r^{-1} + mg$

Tension is **greatest at the bottom of the circle**

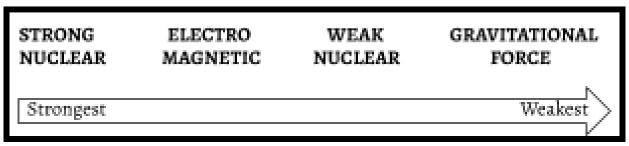
Minimum velocity at the top and bottom of a vertical circular path



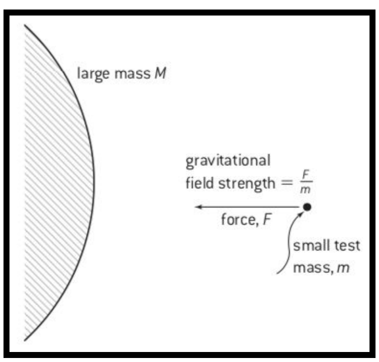
At the top	**At the bottom**
Weight = Centripetal Force	$GPE_T + KE_T = GPE_B + KE_B$
$mg = mv^2r^{-1}$	$mg2r + 0.5m((\sqrt{(gr)})^2) = 0 + 0.5mv^2$
$g = v^2r^{-1}$	$2gr + 0.5gr = 0.5v^2$
$v^2 = gr$	$v^2 = 4gr + gr$
$v = \sqrt{(gr)}$	$v = \sqrt{(5gr)}$

6.2 Newton's Laws of gravitation

Gravitational force is the weakest force in nature:

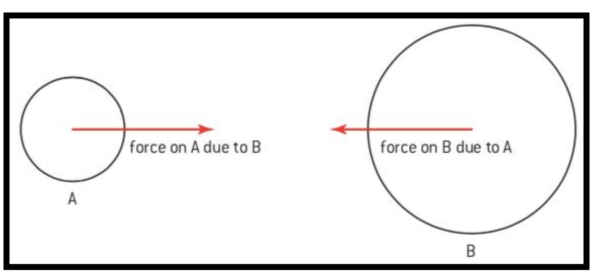


All masses exert a force on each other. Masses that are significantly larger than others simply appear to be attracting smaller masses as their gravitational force is significantly larger.



The **gravitational field strength at a point is the force per unit mass experienced by a small test mass placed at a point in the field**. If the mass of the test mass is m and the field is producing a gravitational force of F, the gravitational field strength, g, is given by $\mathbf{g = Fm^{-1}}$

Cases with two-point masses that attract each other



Newton's Law of Gravitation states that the force **of attraction** between **two point masses** is directly proportional to the product of the masses and inversely proportional to the square of the distance between them.

$$F \propto \frac{Mm}{r^2}$$

To use this equation numerically, a constant G is required, which is equal to 6.67 x 10^{-11}
Therefore,

$$F = \frac{GMm}{r^2}$$

If g = Fm^{-1}, then g also = GMr^{-2}

<u>Kepler's Third Law</u>
The square of the time of revolution of the planet (i.e. their periodic time T) about the sun is proportional to the cubes of their mean distance, r, from it.

$T^2 \propto R^3$

$T^2 = kR^3$

Deriving 'k'

$F_C = F_G$

$mv^2R^{-1} = GMmR^{-2}$

$v^2 = GMR^{-1}$

$(2\pi RT^{-1})^2 = GMR^{-1}$

$4\pi^2R^2T^{-2} = GMR^{-1}$

$T^2 = (4\pi^2(GM)^{-1}) \times R^3$

Therefore, $k = (4\pi^2(GM)^{-1})$

10.1 Describing Fields (Gravitation part only)

A gravitational field exists between two objects that have mass. Each mass has a gravitational field, and any other mass in this field has a gravitational field acting on it. **Gravitational force (F)** is a force that attracts any two objects with mass. **It is always attractive.** This means that the direction of field strength is always towards the mass that gives rise to the field.

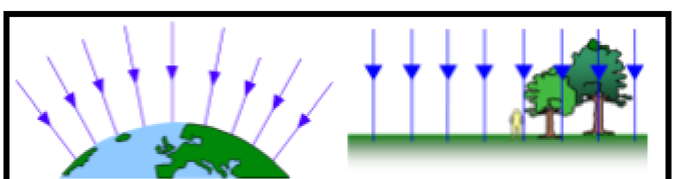
<u>Gravitational field strength</u>
Gravitational field strength is defined as the force per unit mass acting on a small test mass at a point in the field. It is given by the equation: \

$$g = \frac{F}{m}$$

The units of field strength are therefore N kg^{-1}

<u>Field and equipotential in gravitation</u>
Properties of gravitational field lines include:

- Radial field
- Closer the lines, the stronger the field
- Attractive



Gravitational field lines from far

Gravitational field lines from close (Appears like a flat surface)

Gravitational potential

Gravitational potential is the work done per unit mass in bringing a test mass from infinity to a point in the field. It is given by the equation:

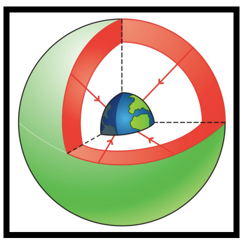
$$\Delta V_g = \frac{W}{m}$$

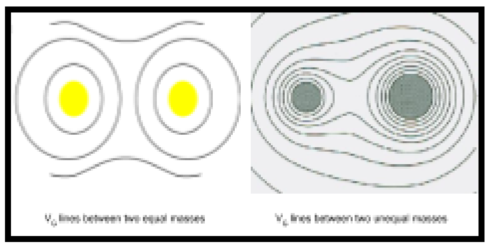
To calculate gravitational potential, **infinity** is taken as a reference point. **Gravitational potential is defined to be zero at infinity**. Newton's Law of gravitation tells us that force is directly proportional to $1/r^2$, so if two masses are an infinite distance apart, there is no interaction between them. F = 0.

As the masses begin to approach each other from infinite separation, they start to become attracted to each other. This means that positive work must be done on the system to push the masses away from each other and back to infinity. Therefore, whenever the masses are closer than infinity (**where gravitational potential is maximum**), the system of the two masses has a **negative potential**.

Equipotential surfaces

Gravitational potential is the same throughout each of those spheres. Work done in moving within an equipotential is **0J. Lines get further apart as you go away.** Furthermore, equipotential lines are **perpendicular** to gravitational field lines.





10.2 Fields at work (Gravitation part only)

Force and inverse-square law behavior

$$F_G = G\frac{m_1 m_2}{r^2}$$

This is Newton's Law of Gravitation (Defined earlier). G is constant and equal to 6.67×10^{-11}

Gravitational potential (continued)

Gravitational field strength is equal to gravitational potential over the radius.

$$g = -\frac{\Delta V_g}{\Delta r}$$

Another equation for the gravitational potential can be derived from Newton's Law of gravitation.

$F = -GMmr^{-2}$

$GPE = F \times d = (-GMmr^{-2}) \times r = -GMmr^{-1}$

$GP = GPE/m = (-GMmr^{-1})/m = -GMr^{-1}$

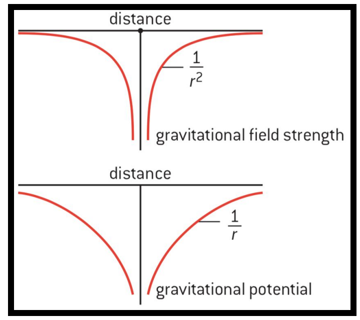
$$V_g = -\frac{GM}{r}$$

Step 2 of the derivation above also shows us that gravitational potential energy is given by:

$$E_p = mV_g = -\frac{GMm}{r}$$

Gravitational potential energy is the work done to move a point mass from infinity to a point in the field. These equations offer graphs that show that as the distance between two masses approaches infinity, the gravitational potential and gravitational potential energy start to reach their maximum of zero.

The area under the Force-distance graph is equal to potential energy
The area under the field strength-distance graph is potential



4 important concepts

Gravitational Quantity	Definition	Unit	Equations	Links to other equations
Force (F)	A force that attracts any two objects with mass	N	$F = G\frac{Mm}{r^2}$	Linked to field strength $g = \frac{F}{m}$
Field strength (g)	The force on unit mass acting on a test mass at a point in the field	N/kg	$g = \frac{F}{m}$	Linked to force Also linked to potential $g = -\frac{\Delta V_g}{\Delta r}$
Potential energy (W)	The work done to move a point mass from infinity to a point in the field	J	$E_p = -\frac{GMm}{r}$	Linked to potential. Divide potential energy by mass to get potential
Potential (V_g)	The work done in moving a small unit mass from infinity to a point in the field	J/kg	$V_g = -\frac{GM}{r}$	Linked to potential energy Linked to field strength

Orbiting

We can determine the orbital velocity around any body.

$$v_{orbit} = \sqrt{\frac{GM}{r}}$$

$F_G = F_C$

$GMmR^{-2} = mv^2r^{-1}$

$V^2 = GMR^{-1}$

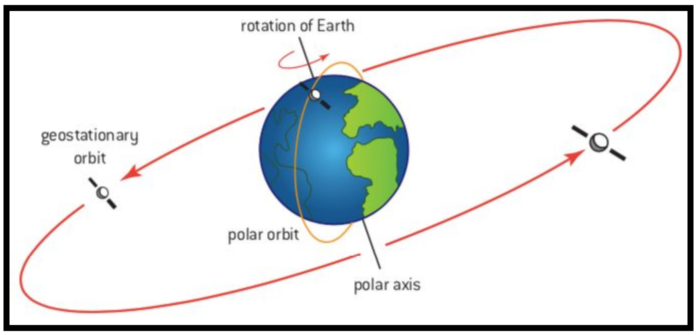
$V = \sqrt{(GMR^{-1})}$

This shows that the mass of the object in orbit does not impact orbital velocity. It is the mass of the larger body, and the distance between the centers of the two bodies that determine the orbital velocity

Types of orbits

There are two useful types of orbits in which satellites are placed (in fact, they can be placed pretty much anywhere). The **polar orbit** is used for satellites closer to the earth (less than 100km). It is called polar, as satellites are usually put into orbit over the poles. Imagine the earth rotating horizontally, and the satellites orbiting vertically (over the poles). This would allow the satellite to view every point on the earth in a 24 period. The other type of orbit is a **geosynchronous orbit**. These are at much larger distances from the earth. Their orbital times are equal to 24 hours, which means these satellites can be made to stay over one part of the sky. They tend to follow a figure of 8 paths over the given region.

A **geostationary orbit** is a special case of the geosynchronous orbit where the satellite is placed in orbit above the plane of the equator. It will not appear to move if viewed from the surface. These make antennae communications easier.



Gravitational potential gradient is the change in gravitational potential per unit distance ($\Delta V_G / \Delta r$)

Escaping the earth

The **escape speed** is the minimum speed an object needs to escape a planet's gravitational pull.

OR

The minimum speed which will carry an object to infinity and bring it to rest there

Escape speed derivation

$\Delta E_K + \Delta E_P = 0$ {Law of conservation of energy}

$\{0.5mv_{final}^2 - 0.5mv_{escape}^2\} + \{(-GMm/r_{final}) - (-GMm/r_{body})\} = 0$

$-0.5mv_{escape}^2 + GMm/r_{body} = 0$ $\{0.5mv_{final}^2 = 0$, as v_{final} is 0 /// $-GMm/r_{final} = 0$, as $r_{final} = \infty\}$

$0.5mv_{escape}^2 = GMm/r_{body}$

$0.5v_{escape}^2 = GMR^{-1}$

$v_{escape} = \sqrt{(2GMR^{-1})}$

Escape speed, therefore, depends on the mass of the planet and radius of the planet, but NOT the mass of the object

Orbital energy

For kinetic energy

$F_G = F_C$

$GMmR^{-2} = mv^2r^{-1}$

$GMmR^{-1} = mv^2$

$0.5GMmR^{-1} = 0.5mv^2$

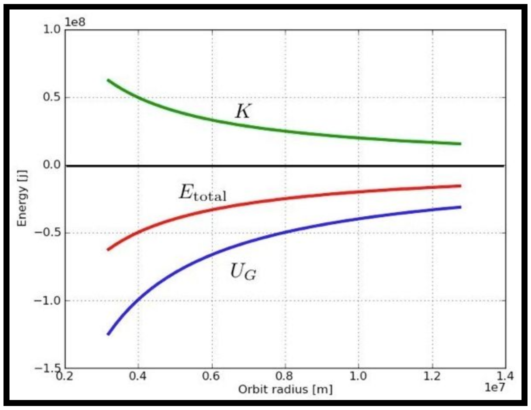
$$KE = \frac{GMm}{2R}$$

For total energy

Total energy = KE + PE

$= 0.5GMmR^{-1} - GMmR^{-1} = -0.5GMmR^{-1}$

Graphs of KE, PE, and Total energy against radius



Orbit shapes

Depending on the speed at which a body is launched from the earth's surface, it follows a certain type of orbital path: